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Information-based trade ☆

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Abstract

We study the possibility of trade for purely informational reasons. We depart from previous analyses (e.g. Grossman and Stiglitz (1980) [22] and Milgrom and Stokey (1982) [32]) by allowing the final payoff of the asset being traded to depend on an action taken by its eventual owner. We characterize conditions under which equilibria with trade exist.

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1. Introduction

Following Grossman and Stiglitz [22] and Milgrom and Stokey [32], economists have reached a consensus that under many circumstances trade between individuals based solely on informational differences is impossible.[1] This result is often described as the "no trade" or "no speculation" theorem. The underlying argument is, at heart, straightforward. If a buyer is prepared to buy an asset from a seller for price p, then the buyer must believe that, conditional on the seller agreeing to the trade, the asset value exceeds p in expectation. But conversely, knowing this the seller is at least as well off keeping the asset. Perhaps motivated by a perception that information-based trade does in fact occur, a large subsequent literature has explored conditions under which the "no trade" conclusion does not hold. For example, Morris [34] and Biais and Bossaerts [6] show that information-based trade is possible without common priors[2]; Dow et al. [16] and Halevy [24] show that information-based trade is possible under non-standard preferences; and the finance market microstructure literature introduces "noise traders" to allow for information-based trade.[3]

In this paper we show that even under completely standard assumptions the no-trade theorem need not hold in a production economy.[4] In particular, in many cases the holder of an asset must make a decision that affects its value. If better information leads to superior decisions, then the information released in trade is socially valuable. This possibility, which is implicitly ruled out in existing no-trade theorems, and neglected by the subsequent literature, is enough to generate information-based trade.

1.1. An example

The intuition for our results is best illustrated by an example.[5] A risk neutral agent (the *seller*) owns an asset that he can potentially trade with a second risk neutral agent (the *buyer*). The asset's payoff depends on two factors: an underlying but currently unobservable fundamental $\theta \in \{a, b\}$, and what the eventual asset owner chooses to do with the asset. If $\theta = a$, the right action to take is A, while if $\theta = b$ the right action is B. The asset pays 1 if the (θ-contingent) right action is taken, and 0 otherwise. The buyer and seller have the same "skill" in taking actions A and B, so that the action-contingent asset payoffs for *both* parties are as given above. One possible interpretation is that the asset is a debt claim and the action is a restructuring decision (e.g., liquidation vs. reorganization). Similarly, the asset may be a controlling equity stake in a firm.[6]

Both the buyer and seller receive private and partially informative signals about the true fundamental θ. Specifically, if $\theta = a$ then each agent observes a signal s^a with certainty, while if $\theta = b$ each agent has a strictly positive probability of observing either s^a or s^b. Consequently, an

[1] See also Jaffe and Rubinstein [28, Corollary to Theorem 2], Kreps [29], Tirole [40], and Fudenberg and Levine [19].

[2] Without common priors, Milgrom and Stokey's belief concordancy condition need not hold.

[3] See Kyle [30] and Glosten and Milgrom [20]; and Dow and Gorton's [15] recent survey.

[4] A number of classic papers (notably, Hirshleifer [25]) note the distinction between information in an exchange economy and information in a production economy. However, the literature on the possibility of trade between differentially and privately informed parties has focused almost exclusively on information in an exchange economy.

[5] The example that follows is a modified version of one that appeared in earlier versions of the paper. We thank an anonymous referee for suggesting the change.

[6] Less extreme, the asset may simply be a large but non-controlling block of shares or debt claims: what matters is that the asset-owner has some control over the decision, rather than absolute control.

agent who observes s^b knows the fundamental is $\theta = b$, while an agent who observes s^a attaches strictly positive probability to both $\theta = a$ and $\theta = b$. For this example, we assume only that the probability of $\theta = a$ given that the seller observes s^a is above $1/2$,[7] and that the probability of $\theta = a$ given that both agents observe s^a is in turn strictly greater.[8]

Fix a price p that lies between these two probabilities, and consider the following trading game: after observing their signals, the buyer and seller simultaneously announce whether they are prepared to trade the asset at the (exogenously fixed) price p. We claim the following is an equilibrium: the buyer offers to buy independent of his signal, and the seller offers to sell if and only if he observes signal s^a.

First, consider the situation faced by the seller. If he ends up with the asset, he must decide what to do using only his own information. On the one hand, if he sees signal s^b and does not sell, he knows $\theta = b$ and takes action B for a payoff of 1, which exceeds the price p offered. On the other hand, if he sees signal s^a and does not sell, he thinks the probability of fundamental a is above $1/2$, and so takes action A; in this case, his expected payoff is less than the price p offered. Consequently, he offers to sell if and only if he sees s^a.

Second, consider the situation faced by the buyer. On the one hand, if he sees signal s^b, he knows $\theta = b$ and hence that he should take action B. He values the asset at 1, which exceeds the price p. On the other hand, if he sees signal s^a, then he combines this with the equilibrium information that the seller observes s^a whenever the buyer actually has a chance to acquire the asset. Consequently, the buyer takes action A if he acquires the asset after seeing s^a, since he thinks the probability of $\theta = a$ is above $1/2$. Moreover, the buyer's valuation of the asset exceeds the price p. Hence the buyer offers to buy regardless of his signal.

Note that both parties are strictly better off under the trade, even after conditioning on any information they acquire in equilibrium. This is exactly what Milgrom and Stokey prove is impossible in their framework, and the reason it is possible here is that the asset value endogenously depends on the information possessed by its owner. In contrast, in papers such as Grossman and Stiglitz, and Milgrom and Stokey, asset holders have no decision to make since the final asset payoffs are exogenous to the information possessed by its owner.

In the example, trade transfers the asset from the seller when he observes signal s^a to the buyer. Trade creates value because it leads to a better decision after the seller sees s^a and the buyer sees s^b. Specifically, after these signals the seller would take action A because he observes only signal s^a; while the right action is B, and the buyer takes this action. In essence, trade transfers the asset from an agent who is likely to make the wrong decision to one who is more likely to make the right decision.

The feature that the buyer sometimes takes a different action than the seller would have taken if he (counterfactually) retained the asset is a general one — see part (iii) of Theorem 1 below. In this sense, trade is necessarily associated with a change in action. Two possible applications include the role of vulture investors in debt restructuring, and corporate raiders. With regard to the former, it is widely perceived that vulture investors' behavior in restructuring negotiations differs from that of the original creditors (see, e.g., Morris [33]). With regard to the latter, there is evidence that large scale layoffs and divestitures follow takeovers (see, e.g., Bhagat et al. [5]).

7 This is necessarily the case if the two possible realizations of θ are equally likely.

8 This is necessarily the case if the two agents' signals are independent conditional on θ.

1.2. Discussion

One way to view our analysis is as saying that existing no-trade results have a smaller domain of application than is traditionally believed. Specifically, to separate trade for informational reasons from trade for preference reasons, papers such as Milgrom and Stokey impose a requirement that the initial allocation is *ex ante* Pareto efficient. To ensure there is no preference motive for trade in our framework, we assume that all agents are risk-neutral. So if one defines an allocation simply as specifying who owns the asset, along with a distribution of the numeraire good, any initial allocation is Pareto efficient.

On the other hand, one could instead define an allocation in our framework as a mapping from the set of signals agents receive (e.g., $\{s^a, s^b\} \times \{s^a, s^b\}$ in the example) into decisions (e.g., A vs. B) and transfers of the asset and numeraire good. This is what Holmström and Myerson [26] refer to as a "decision rule." The initial "decision rule" in our example (i.e., seller keeps the asset and takes a decision based only on his own information) is Pareto inefficient. Hence using this definition the Pareto efficiency requirement rules out too much, as our analysis demonstrates: trade is necessarily for information reasons, since there is no preference motive for trade.

Returning to Holmström and Myerson, they study a wider class of economies than those covered by no-trade papers: agents' actions may affect output, as in our model. They establish the following mechanism-design result: if a "decision rule" is *ex ante* Pareto efficient out of incentive-compatible allocations, it is still Pareto efficient out of incentive-compatible allocations at the interim stage. However, in common with most mechanism-design papers, Holmström and Myerson are silent on the actual mechanism that delivers the *ex ante* Pareto efficient allocation. The contribution of our paper is to show that trade — a widely observed mechanism that has long interested economists, and has many appealing features — is a mechanism capable of improving upon the status quo allocation in which the seller keeps the asset and takes a decision based only on his own information. Indeed, in our example trade results in the first-best outcome.[9]

1.3. Paper outline

In Section 2 we review related papers not already discussed. In Section 3 we present the general model, which closely resembles the example above but with the binary action set and signal space replaced with an arbitrary action set and continuous signal space.

In Section 4 we establish a simple necessary and sufficient condition trade. Of course, in general the conditions under which trade is possible depend to some extent on the institutional environment. However, it is also clear that we want our results to be as independent as possible of *a priori* assumptions about the trading environment. To meet these objectives, we establish a necessary condition for trade to occur in a very wide class of trading mechanisms. Specifically, we follow the literature and ask whether it is possible for there to be common knowledge of gains from trade. We then show that our necessary condition is also sufficient for at least one simple trading mechanism, in which the price is exogenously set.

[9] This is not true in our more general model below. This raises the question of what the optimal mechanism is. Absent information acquisition costs, there is a simple mechanism that has the first-best outcome as an equilibrium: simply ask the buyer to announce his signal, and let the seller keep the asset. However, clearly this simple mechanism fails with even a small cost of information acquisition. In contrast, we show in Proposition 1 that agents would pay to acquire their information in the trade mechanism we study. We leave the complicated question of the optimal mechanism with information acquisition costs for future research.

In Section 5 we then consider trade in two settings where the price is endogenous. First, we analyze a trading mechanism in which the buyer makes a take-it-or-leave-it offer. Second, we consider trade in a competitive equilibrium. In Section 6, we consider how our results change if agents can also trade a security whose payoff directly depends on the action eventually taken. Finally, Section 7 concludes with an extended discussion of two remaining issues: an alternative interpretation of our model in which the action is a hedging decision; and the effect of the information used to assess a trade.

2. Related literature

We have already reviewed much of the related literature. To the best of our knowledge the only previous consideration of trade for purely informational reasons in an economy in which asset owners must decide how to use their assets is a chapter of Diamond's [12] dissertation.[10] He derives conditions under which a rational expectations equilibrium (REE) with trade exists when there are two types of agents: one type is uninformed, while the other type observes a noisy signal. The main differences between our paper and his are that (i) we study trade between agents who both possess information, (ii) we show that as a consequence, information is never fully revealed, and (iii) instead of restricting attention to the competitive (REE) outcome, in the spirit of Milgrom and Stokey [32] we allow for all possible trading mechanisms. Moreover, Diamond's assumption that one side of the trade is completely uninformed means that assets always flow from the less to the more informed party.[11] In contrast, when both parties to the trade have some information, assets can flow to the party with lower quality information.

Our paper is also related to a growing "feedback" literature dealing with models in which investors trade with the understanding that the equilibrium price affects real decisions and hence the profitability of their trades: see Dow and Gorton [14] for an early example, and Goldstein and Guembel [21] or Bond et al. [8] for more recent contributions. However, the models in this literature rely on noise-traders to generate trade, and say nothing about the possibility of trade when there is no exogenous source of noise.

3. The model

Our model is closely related to the opening example. As in the example, there are two risk neutral agents, who we refer to as a seller (agent 1) and a buyer (agent 2).[12] The seller owns an asset. The payoff from the asset depends on the combination of the action taken by the asset-owner and the realization of some fundamental $\theta \in \{a, b\}$. Neither agent directly observes the fundamental θ, but before meeting, both agents $i = 1, 2$ receive noisy and partially informative

[10] Less closely related is a recent working paper of Tetlock and Hahn [39], who show that a decision maker would be willing to trade and act as a loss-making market maker in "weather" securities (or more generally, securities whose value is exogenous to the decision).

[11] Diamond does consider an equilibrium in which uninformed agents end up holding the asset. However, to support the equilibrium he must assume that uninformed agents learn only from the price at which the trade takes place, and not from the volume of trade.

[12] Our analysis can be extended to cover risk-averse agents with constant absolute risk aversion preferences (see Section 7). However, the risk-neutrality assumption has the advantage of making clear that any trade is for information reasons.

signals s_i. Whereas in the example signals were binary, in our main model they have full support in $\mathbb{R}$.[13]

The eventual asset owner must decide what action to take. Regardless of whether the asset-owner is agent 1 or 2, the range of available actions is given by a compact set $\mathcal{X}$, with a typical element denoted by X. (In the opening example, $\mathcal{X}$ is simply the binary set $\{A, B\}$.) We write $v(X, \theta)$ for the payoff when action X is taken and the fundamental is θ, where $v(\cdot, \theta)$ is continuous as a function of X. We emphasize that the asset payoff is independent of the identity of the asset-owner — both agents 1 and 2 are equally capable of executing all actions in $\mathcal{X}$.

3.1. Pre-trade information

The information structure of the economy is described by a probability measure space $(\Omega, \mathcal{F}, \mu)$, where $\Omega = \{a, b\} \times \mathbb{R}^2$ and $\mathcal{F}$ is the σ-algebra $\{\{a\}, \{b\}, \{a, b\}\} \times \mathcal{B}^2$. (Throughout, we denote the Borel algebras of $\mathbb{R}$ and $\mathbb{R}^2$ by $\mathcal{B}$ and $\mathcal{B}^2$ respectively.) We write a typical state as $\omega = (\theta, s_1, s_2)$, where θ is the fundamental, s_1 is the signal observed by the seller (agent 1) and s_2 is the signal observed by the buyer (agent 2). For $i = 1, 2$ and $\theta = a, b$ let $\eta_i^\theta : \mathcal{B} \to \mathbb{R}$ be the conditional distribution of s_i given θ. We write F_i^θ and f_i^θ respectively for the associated distribution and density functions, which we assume to be continuous, and make the following additional assumptions. (I) The signals s_1 and s_2 are conditionally independent given θ.[14] (II) For $i = 1, 2$ and $\theta = a, b$ the conditional distribution η_i^θ has full support. (III) For $i = 1, 2$ signal s_i satisfies the strict monotone ratio likelihood property (MLRP), i.e., $L_i(s_i) \equiv \frac{f_i^a(s_i)}{f_i^b(s_i)}$ is strictly increasing in s_i. Moreover, we assume that the likelihood ratio is unbounded, i.e.,

$$L_i(s_i) \to 0, \infty \quad \text{as } s_i \to -\infty, +\infty. \tag{1}$$

That is, there are extreme realizations of each agent's signal that are very informative — even if an agent's signal is generally uninformative. (We stress that none of the necessary conditions established in our main result, Theorem 1, depend on the assumption of unbounded likelihood ratios. See also the discussion on page 1685.)

Agent i directly observes only his own signal. Formally, the information of agents $i = 1, 2$ before trade is given by the sub-σ-algebras $\mathcal{F}_1 = \{a, b\} \times \mathcal{B} \times \mathbb{R}$ and $\mathcal{F}_2 = \{a, b\} \times \mathbb{R} \times \mathcal{B}$.

3.2. Trade

An allocation in our economy is a pair of mappings $\kappa : \Omega \to \{1, 2\}$ and $\pi : \Omega \to \mathbb{R}$ where κ specifies which agent owns the asset, and π specifies a transfer from agent 2 to agent 1. Since neither agent observes the fundamental θ both κ and π must be measurable with respect to the σ-algebra $\{a, b\} \times \mathcal{B}^2$. Let $(\hat{\kappa}, \hat{\pi})$ denote the initial allocation, in which agent 1 owns the asset

[13] We use the assumption that signals are drawn from a continuum to establish the general existence of an equilibrium with trade. Specifically, the assumption allows us to use standard fixed-point arguments for continuous functions.

[14] We use conditional independence in two places in the proof of our main result, Theorem 1. First, in the proof of part (ii), the assumption is used to evaluate inequality (26). Here, it would be straightforward to substantially weaken the assumption of conditional independence. Second, in the proof of the sufficiency half of part (i), the assumption is used to show that the system of Eqs. (15)–(18) has a solution. However, we stress that conditional independence is a sufficient but not necessary condition for common knowledge of gains from trade to exist. This is illustrated by the example in the introduction, where conditional independence is not assumed.

and no transfer takes place: $(\hat{\kappa}, \hat{\pi}) \equiv (1, 0)$. A *trade* is simply an allocation (κ, π) that differs from the initial allocation with strictly positive probability.

3.3. Post-trade information

After trade, agent i's information is given by a σ-algebra $\mathcal{F}_i^{\kappa,\pi} \subset \mathcal{F}$, where $\mathcal{F}_i \subset \mathcal{F}_i^{\kappa,\pi} \subset \{a, b\} \times \mathcal{B}^2$. That is, each agent remembers his own signal, and learns at most the other agent's signal.

Each agent observes the outcome of the trade, and updates his information accordingly. Formally, κ and π are $\mathcal{F}_i^{\kappa,\pi}$-measurable. Moreover, in principle it is possible that the trade mechanism entails the release of additional information to agent i. In this case, the σ-algebra generated by (κ, π) would be a strict sub-algebra of $\mathcal{F}_i^{\kappa,\pi}$.

An important object in our analysis is the probability that an agent attaches to fundamental a (or b) conditional on some information. Notationally, for any σ-algebra $\mathcal{G}$ let $Q(\omega; \mathcal{G})$ denote the conditional probability of $\{a\} \times \mathbb{R}^2$ in state ω relative to $\mathcal{G}$.

3.4. Endogenous asset values

The eventual asset owner must select an action $X \in \mathcal{X}$ without knowing the realization of fundamental θ. For each candidate action X he can evaluate the expected payoff under that action. We denote this expected payoff by $V(q; X) \equiv qv(X, a) + (1 - q)v(X, b)$, where q denotes the probability the agent places on fundamental a. Since the asset owner chooses the action with the highest expected payoff, his valuation of the asset is given by

$$V(q) \equiv \max_{X \in \mathcal{X}} V(q; X). \tag{2}$$

Note that V is continuous over [0, 1], and hence bounded.[15] Moreover, because V is the upper envelope of linear functions it is convex. Economically, the convexity of V corresponds to the fact that information is valuable because it allows the asset-holder to select an action better suited to the fundamental θ.

3.5. Common knowledge of gains from trade

As discussed on page 1678, we follow the literature and ask whether it is possible for there to be common knowledge of gains from trade, with the gains from trade strictly positive with strictly positive probability. Formally, if the seller and buyer's information is given by $\mathcal{G}_1$ and $\mathcal{G}_2$

[15] We have assumed that the fundamental θ is binary-valued. The significance of this assumption is that it allows us to define the asset value V as a function of a one-dimensional summary statistic, namely the probability q that the fundamental is a. That is, uncertainty is unidimensional. Unidimensionality greatly facilitates the derivation of sufficient conditions for trade in Theorem 1. We conjecture the necessary conditions of Theorem 1 would extend to more general state spaces.

It should also be noted that it is possible to obtain a similarly tractable unidimensional framework with a richer set of fundamentals, though at the cost of introducing more assumptions on the asset payoff functions $v(X, \theta)$. For example, one could allow the fundamental θ to be drawn from an arbitrary subset of $\mathbb{R}$, but restrict the asset payoff to take the form $v(X, \theta) = K(X) + M(X)\theta$ for an arbitrary pair of continuous functions K and M. In this case, the expected asset payoff given action X is a linear function of the expected value of θ, and so one can define an analogous function to V that depends only on a one-dimensional variable (i.e., the expected value of θ as opposed to the probability of a).

respectively, an event D is common knowledge if it differs from a member of $\mathcal{G}_1 \cap \mathcal{G}_2$ by a null set (Nielsen [35]). Consequently, evaluated using information $\mathcal{G}_1$ and $\mathcal{G}_2$, there exists common knowledge of strict gains from trade if there exists $D \in \mathcal{G}_1 \cap \mathcal{G}_2$ such that for almost all $\omega' \in D$, both the seller and buyer are better off in expectation, i.e.,

$$E_\omega\big[\pi(\omega) + \mathbf{1}\big(\kappa(\omega) = 1\big)V\big(Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big)\big|\mathcal{G}_1, \omega'\big] \geqslant E_\omega\big[V\big(Q(\omega; \mathcal{G}_1)\big)\big|\mathcal{G}_1, \omega'\big],$$
$$E_\omega\big[-\pi(\omega) + \mathbf{1}\big(\kappa(\omega) = 2\big)V\big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big)\big)\big|\mathcal{G}_2, \omega'\big] \geqslant 0,$$

and such that there exists a non-null subset $D' \in \mathcal{F}$ of D such that for almost all $\omega' \in D'$, at least one of the two inequalities holds strictly.

For most of the remainder of the paper we assume that the buyer and seller use their post-trade information, $\mathcal{F}_1^{\kappa,\pi}$ and $\mathcal{F}_2^{\kappa,\pi}$, to evaluate a trade, and so common knowledge is evaluated using the σ-algebra $\mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$. We make this assumption for two reasons. First, having agents evaluate a trade using relatively fine information biases our analysis against finding common knowledge of gains from trade. Second, the assumption is used in prior analyses: in particular, it is used in analyses based on competitive equilibria (e.g., Kreps [29]), and (at least in our reading) it is used by Milgrom and Stokey [32].[16] Nonetheless, in Section 7 below we discuss how this assumption affects our results, and consider the case in which the buyer and seller instead evaluate a trade using their pre-trade private information (i.e., $\mathcal{F}_1$ and $\mathcal{F}_2$).

Setting $\mathcal{G}_i = \mathcal{F}_i^{\kappa,\pi}$, and using the fact that π and κ are measurable with respect to $\mathcal{F}_1^{\kappa,\pi}$ and $\mathcal{F}_2^{\kappa,\pi}$, we obtain:

Definition 1. There exists common knowledge of strict gains from trade if there exists $D \in \mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$ such that for almost all $\omega \in D$,

$$\pi(\omega) + \mathbf{1}\big(\kappa(\omega) = 1\big)V\big(Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big) \geqslant V\big(Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big), \tag{3}$$
$$-\pi(\omega) + \mathbf{1}\big(\kappa(\omega) = 2\big)V\big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big)\big) \geqslant 0, \tag{4}$$

and such that there exists a non-null subset $D' \in \mathcal{F}$ of D such that for almost all $\omega \in D'$, at least one of the two inequalities holds strictly.

The above definition covers both states in which the asset changes hands ($\kappa(\omega) = 2$) and those in which the seller keeps the asset ($\kappa(\omega) = 1$). In the latter case, inequalities (3) and (4) imply that in almost all states that the seller keeps the asset, no money is transfered ($\pi(\omega) = 0$). This observation allows us to simplify the condition for common knowledge of strict gains from trade:

Lemma 1. *There exists common knowledge of strict gains from trade if and only if there exists $D \in \mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$ such that the buyer acquires the asset for all $\omega \in D$, and for almost all $\omega \in D$,*

$$V\big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big)\big) \geqslant \pi(\omega) \geqslant V\big(Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big), \tag{5}$$

and there exists a non-null subset $D' \in \mathcal{F}$ of D such that for almost all $\omega \in D'$, at least one of the two inequalities holds strictly.

All proofs are in Appendix A.

[16] In Milgrom and Stokey, agent i evaluates the trade according to "his information at the time of trading, including whatever he can infer from prices or from the behavior of other traders" (p. 19). As Dow et al. [16] and Rubinstein and Wolinsky [36] make clear, however, Milgrom and Stokey's result does not depend on the information used to evaluate a trade: see Section 7 below.

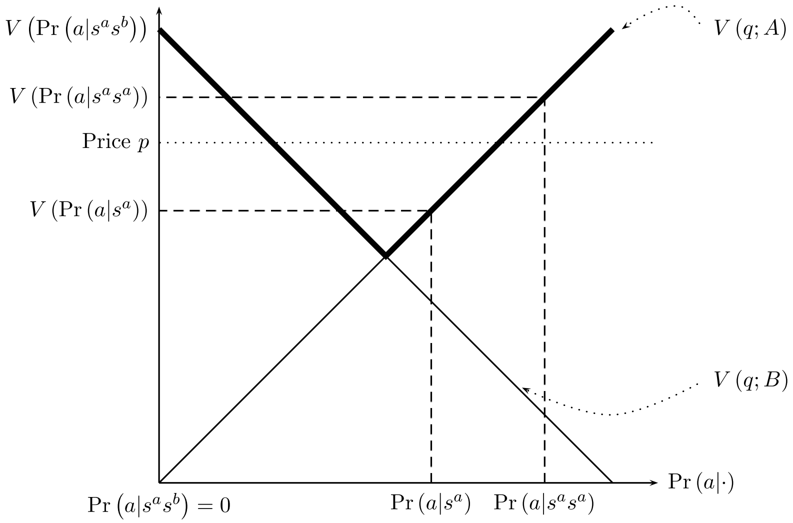

Fig. 1. The graph displays $V(q;X)$ for the opening example: the action set is $\mathcal{X} = \{A, B\}$ and both the buyer and seller observe signals drawn from $\{s^a, s^b\}$. The bold line is the upper envelope of these two functions, and corresponds to the function $V(q)$.

4. Analysis

In this section we establish necessary and sufficient conditions for the existence of common knowledge of strict gains from trade.

It is useful to first reconsider the opening example, which is displayed graphically in Fig. 1. Recall that in the example trade occurs whenever the seller sees signal s^a. The buyer's valuation when trade occurs is driven by the probability he places on fundamental a, i.e., $\Pr(a|s^a s^a)$ or $\Pr(a|s^a s^b)$, depending on his own signal realization. Trade is possible because the value of the asset given these probabilities exceeds the value of the asset when the probability of fundamental a is $\Pr(a|s^a)$, which is the information the seller has.

The key feature of the example that determines how the value of the asset varies across the probabilities $\Pr(a|s^a s^a) > \Pr(a|s^a) > \Pr(a|s^a s^b)$ is that the asset is worth more if the realization of θ is known for sure than if there is some small uncertainty about its realization, *regardless of whether the realization of θ is a or b*. In other words, more precise information is better.

This condition can be more formally stated as follows. Define $h(q) \equiv \frac{1}{q(1-q)}$ as the precision (i.e., the reciprocal of variance) of the Bernoulli distribution with success probability q. Then the condition is[17]:

Precision condition (Precision is valuable). *$\exists \bar{h}$ and $\varepsilon > 0$ such that $\forall q, q'$, if $h(q) > h(q') \geqslant \bar{h}$ and $|q - q'| < \varepsilon$, then $V(q) > V(q')$.*

In contrast, if instead the asset-owner has no action choice, then the asset must be more valuable in one fundamental than the other. In this case, the precision condition cannot hold; and the asset value must be greater at $\Pr(a|s^a)$ than one of $\Pr(a|s^a s^a)$ and $\Pr(a|s^a s^b)$, implying that at

[17] Geometrically, the precision condition is equivalent to non-monotonicity of V. See Lemma 2 in Appendix A.

least one of the buyer and seller is made worse off after at least one of the signal pairs $s^a s^a$ and $s^a s^b$.

Our main result establishes that the precision condition is both necessary and sufficient for the possibility of common knowledge of strict gains from trade. Additionally, it characterizes two features that any such trade must exhibit:

Theorem 1.

(i) *Common knowledge of strict gains from trade can exist if and only if the asset value is increasing in the precision of information for sufficiently high levels of precision of information, i.e., the precision condition holds. Whenever common knowledge of strict gains from trade exists,*
(ii) *the buyer sometimes learns something when he acquires the asset, and*
(iii) *there exists a non-null set of states in which the buyer acquires the asset, and in which his action differs from the action the seller would take if* (*counterfactually*) *he controlled the asset in the same state.*

We have already discussed the necessity half of part (i) of the theorem. We now discuss the remainder.

Clearly if a condition is necessary for trade, it must be so under any mechanism while, sufficiency depends, in part, on the trading mechanism used. In showing the sufficiency of the precision condition for trade, we use a constructive proof for arguably the simplest mechanism possible: a non-strategic third-party — a "broker" — sets a price p, and then the buyer and seller simultaneously and publicly announce whether they wish to trade at price p. The proof shows that this mechanism allows trade for any price p above $\min V$ but below $\min\{V(0), V(1)\}$ (note that under the precision condition the function V has an interior minimum).

Specifically, the proof shows that for any such price p there exists an equilibrium of the following form: the seller offers to sell when he sees a signal $s_1 \in S_1^T \equiv [\underline{s}_1, \bar{s}_1]$, and the buyer offers to buy if he sees a signal $s_2 \in S_2^T \equiv \mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)$. In this equilibrium, the buyer offers to buy whenever his signal is either high or low, that is, when it is relatively informative of the fundamental. Under the precision condition, the buyer's valuation of the asset is relatively high at such signals because the precision of information is high. Similarly, the seller offers to sell when he sees an intermediate signal, that is, a signal that is relatively uninformative about the fundamental. Given the precision condition, the seller's valuation is relatively low at such signals.

Our proof establishes the existence of a continuum of equilibria, indexed by the trade price p. Comparing the lowest price $p = \min V$ to the highest price $p = \min\{V(0), V(1)\}$, the buyer's demand (i.e., the probability of accepting the price) decreases (from 1 to 0), while the seller's supply increases (from 0 to 1). That is, the comparative static across equilibria generates a downwards sloping demand curve and an upwards sloping supply curve.[18]

In equilibrium, trade transfers control of the asset from an agent who has received an uninformative signal to one who has received an informative signal. Moreover, because of its contingent nature trade also reveals information about the agents' signals to each other. Specifically, when the seller retains the asset he learns whether or not the buyer's signal is in S_2^T; and when the

[18] Because of the interdependency between the buyer and seller, more conditions would be required to establish that the demand (respectively, supply) curve is *monotonically* downwards (respectively, upwards) sloping.

buyer acquires the asset, he learns that the seller's signal is in S_1^T. The fact that buyer learns something in equilibrium is a more general property that must hold under any mechanism. Part (ii) of Theorem 1 shows that trade is not possible if it does not convey some information to the buyer. This conclusion is very much in line with the existing no-trade literature. At the same time, and as part (i) makes clear, trade is at least sometimes possible if it enables the buyer to learn the seller's signal.

To understand this point, it again helps to reconsider the opening example. Suppose that trade occurred in this example *without* the buyer learning anything about the seller's signal. For specificity, suppose further that trade only occurs when the buyer sees signal s^a.[19] Recall that agents always learn at least the information revealed by the trade allocation. Consequently, for the buyer not to learn anything trade must occur after both signal pairs $s^a s^a$ and $s^b s^a$. So when trade occurs, the buyer places probability $\Pr(a|s^a)$ on the fundamental being a; while the seller places probability $\Pr(a|s^a s^a)$ or $\Pr(a|s^b s^a)$ on the fundamental being a, depending on his own signal realization. Because information is valuable the asset value V is convex — see the discussion following (2). Consequently, the asset value at at least one of $\Pr(a|s^a s^a)$ and $\Pr(a|s^b s^a)$ must exceed the asset value at $\Pr(a|s^a)$. But then the seller's valuation exceeds the buyer's valuation in at least one of the signal realizations $s^a s^a$ and $s^b s^a$, ruling out the possibility of common knowledge of gains from trade.

The proof of part (ii) of Theorem 1 is along the same lines as the above discussion of the example. The main complication in the formal proof is the need to form conditional probabilities for arbitrary information possessed by the seller. At the same time, the proof is simplified somewhat by our assumption of unbounded likelihood ratios (see (1)). We emphasize, however, that (as the example illustrates) this property is not essential for the result, and a proof for the case of finite signal spaces — and hence bounded likelihood ratios — is available on the authors' web-pages.

Finally, consider part (iii) of Theorem 1, which states that trade is associated with a change in action. An immediate implication of Lemma 1 is that common knowledge of strict gains from trade exists only if there exists $D \in \mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$ such that

$$\int_D \left(V\left(Q\left(\omega; \mathcal{F}_2^{\kappa,\pi}\right)\right) - V\left(Q\left(\omega; \mathcal{F}_1^{\kappa,\pi}\right)\right)\right) \mu(d\omega) > 0. \tag{6}$$

Inequality (6) represents the benefits of trade. Since the buyer's information in state ω is different from the seller's, he potentially takes a different action. This causes the value of the asset when owned by the buyer to potentially diverge from the value of the asset owned by the seller, in spite of their equal ability to execute all actions $X \in \mathcal{X}$. Part (iii) of Theorem 1 follows almost immediately from (6). Moreover, note that this conclusion is also a corollary of Milgrom and Stokey's main result.

Finally, as an immediate corollary of part (i) of Theorem 1 we obtain:

Corollary 1. *Common knowledge of strict gains from trade can exist only if* (i) *there is no dominant action, i.e.,* $\nexists X \in \mathcal{X}$ *such that* $v(X,\theta) \geqslant v(X',\theta)$ *for all* $X' \neq X$ *and* $\theta = a, b$; *and* (ii) *there is no dominant fundamental, i.e.,* $\nexists \theta \in \{a,b\}$ *such that* $v(X,\theta) \geqslant v(X,\theta')$ *for all* $X \in \mathcal{X}$ *and* $\theta' \neq \theta$.

[19] Similar arguments apply for the cases of trade following signal s^b, and trade after both buyer signal realizations.

An important implication of the no-trade theorems established in the existing literature is that economic agents would not spend resources to acquire information. In contrast, our next result shows that this is not true in our model.[20] The key reason is, of course, that information is valuable. The non-trivial aspect of the result consists of showing that an agent's information is valuable above-and-beyond the information he acquires from the other agent in the course of trade.

Proposition 1. *Suppose the buyer and seller must each incur a cost $k > 0$ in order to observe their signals. Fix any price $p \in (\min V, \min\{V(0), V(1)\})$. Provided the information acquisition cost k is sufficiently small there exists an equilibrium of the third-party posted price mechanism in which both the buyer and seller acquire their signals and trade occurs with positive probability.*

5. Endogenous prices

The third-party posted price mechanism we considered above describes many trading environments well. For example, both buyers and sellers take the price as exogenous when they submit market orders; in upstairs trades, in which the upstairs broker proposes the price[21]; and in crossing networks (POSIT is a well-known example) in which the price is determined elsewhere. Nonetheless, in many other situations the price is set endogenously.

5.1. Buyer proposes price

One simple trade mechanism in which prices are set endogenously is that in which the buyer makes a take-it-or-leave-it offer to buy the asset for a price p, and the seller either accepts or rejects. We analyze exactly this mechanism in Appendix B,[22] and establish that when the precision condition holds, there are at least some circumstances in which common knowledge of strict gains from trade are possible in the buyer-proposed price mechanism. Moreover, we show that the buyer's proposed price *necessarily* communicates some of the buyer's information to the seller. Specifically, it is impossible to have a meaningful probability of trade in an equilibrium in which trade occurs at only one price. So trade must occur at at least two distinct prices, and which price the buyer proposes reveals information about the buyer's signal to the seller. In equilibrium, the buyer is willing to offer different prices because the higher price is associated with a higher acceptance probability.

5.2. Decentralization

An alternative way to endogenize prices is as part of a competitive equilibrium. Accordingly, we next show that the trade constructed in the sufficiency half of part (i) of Theorem 1 can be decentralized as a competitive equilibrium. Specifically, we show that there is a generalized rational expectations equilibrium (GREE) in which the buyer acquires the asset for price p whenever

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[20] For other papers in which agents spend resources to acquire information prior to trade even in the absence of exogenous noise, see, for example, Berk [4] and Jackson and Peck [27].

[21] Many large trades occur in "upstairs" markets, i.e., are trades in which "buyers and sellers negotiate in the 'upstairs' trading rooms of brokerage firms" (Booth et al. [9]). Identifying upstairs trades is relatively hard, but using detailed data from Finland Booth et al. report that upstairs trades account for 50% of total volume.

[22] We assume that the buyer's offer p is constrained to lie within some finite set P. In other words, the buyer's action set is finite. As is well known, equilibrium existence is not guaranteed in games with infinite action sets.

the signal pair (s_1, s_2) lies in $S_1^T \times S_2^T$, where p, S_1^T, and S_2^T are as in the proof of part (i) of Theorem 1. We discuss below why we are using GREE as our competitive equilibrium concept rather than a REE.

A GREE differs from a REE in that agents condition on information beyond that contained in the price — in particular, in the most standard application a GREE allows agents to condition on volume (Kreps [29]; Allen and Jordan [3]; Schneider [37]). In our setting, volume equals 1 if $\kappa(\omega) = 2$ (the buyer acquires the asset), while volume equals 0 if $\kappa(\omega) = 1$ (the seller keeps the asset). Let $\mathcal{F}^{12}$ denote the σ-algebra relating to the join of the buyer's and seller's information, i.e., $\mathcal{F}^{12} \equiv \{\{a, b\}\} \times \mathcal{B}^2$. Decentralization in a competitive equilibrium requires that agents should be able to buy/sell the quantity they desire at the posted price, and that markets then clear. Formally, then, a GREE in our setting is a pair of $\mathcal{F}^{12}$-measurable functions $p : \Omega \to \mathbb{R}$ and $\kappa : \Omega \to \mathbb{R}$ such that for almost all $\omega \in \Omega$, either

$$\begin{aligned}&\kappa(\omega) = 1 \quad \text{and} \quad V\big(\Pr\big(a \big| s_1, p(\omega), \kappa(\omega)\big)\big) \geqslant p(\omega) \quad \text{and}\\ &V\big(\Pr\big(a \big| s_2, p(\omega), \kappa(\omega)\big)\big) \leqslant p(\omega)\end{aligned} \tag{7}$$

or

$$\begin{aligned}&\kappa(\omega) = 2 \quad \text{and} \quad V\big(\Pr\big(a \big| s_1, p(\omega), \kappa(\omega)\big)\big) \leqslant p(\omega) \quad \text{and}\\ &V\big(\Pr\big(a \big| s_2, p(\omega), \kappa(\omega)\big)\big) \geqslant p(\omega).\end{aligned} \tag{8}$$

Conditions (7) and (8) say that either volume is 0 and the price is such that the seller is happy not to sell (given the information revealed by price, volume, and the seller's own signal) and the buyer is happy not to buy; or that volume is 1 and the price is such that the seller is happy to sell and the buyer is happy to buy.

Let V attain its minimum value over $[\underline{q}, \bar{q}]$, where $\underline{q} = \bar{q}$ if V has a unique minimizer (as observed above, V has an internal minimum under the precision condition). We claim the following is a GREE. If the state ω belongs to the trade set $\{a, b\} \times S_1^T \times S_2^T$, then trade occurs, $\kappa(\omega) = 2$, at the specified price, $p(\omega) = p$. If instead the state ω lies outside the set $\{a, b\} \times S_1^T \times S_2^T$, then the price equals the full information value of the asset, $p(\omega) = V(\Pr(a|s_1, s_2))$, and trade may or may not occur: $\kappa(\omega) = 1$ if $\Pr(a|s_1, s_2) \leqslant \bar{q}$ and $\kappa(\omega) = 2$ if $\Pr(a|s_1, s_2) > \bar{q}$. As we explain below, the buyer and seller are indifferent between trading and not trading in this second region.

Given that p, S_1^T and S_2^T are taken from an equilibrium in the third-party posted price mechanism, it is immediate that when $\omega \in \{a, b\} \times S_1^T \times S_2^T$ condition (8) holds. Moreover, there are strict gains from trade for almost all such states: the buyer strictly prefers to buy, and the seller strictly prefers to sell.

If instead $\omega \notin \{a, b\} \times S_1^T \times S_2^T$, the conjectured GREE features a price and volume that fully reveal all relevant information, namely $\Pr(a|s_1, s_2)$. This follows from the convexity of V, which ensures that any value above $\min V$ is associated with at most two probabilities, $q' < \underline{q}$ and $q'' > \bar{q}$, say. The conjectured GREE entails different volumes for these two probabilities: volume equals 0 if $\Pr(a|s_1, s_2) = q'$ and equals 1 if $\Pr(a|s_1, s_2) = q''$. (If $\Pr(a|s_1, s_2) \in [\underline{q}, \bar{q}]$ then price and volume reveal only that $\Pr(a|s_1, s_2)$ lies in $[\underline{q}, \bar{q}]$, but this is still sufficient to infer that the value of the asset given s_1 and s_2 is $\min V$.) Because price and volume together reveal $\Pr(a|s_1, s_2)$, both the buyer and seller are indifferent between trading and not trading at the price $V(\Pr(a|s_1, s_2))$. Consequently, the conjectured κ and p do indeed constitute a GREE.

We conclude with a discussion of why we have worked with GREE and not REE. There are two reasons. First, our agents are risk-neutral, and so as Kreps [29] observes, there may exist pathological REE in which an agent is unconditionally worse off in equilibrium than if he simply stuck with his original endowment; and these equilibria are ruled out if agents are allowed to condition on volume, as in a GREE.[23]

Second, we need to ensure that markets clear for all $\omega \notin \{a, b\} \times S_1^T \times S_2^T$, and the only way we have found to do this is to make sure that the buyer and seller value the asset equally in these states. In turn, the only way we have found to guarantee equal valuation by the two parties is to have $\Pr(a|s_1, s_2)$ fully revealed in equilibrium in states outside $\{a, b\} \times S_1^T \times S_2^T$. However, because the precision condition implies non-monotonicity of V, price alone is generally insufficient for full revelation. Note that if the signal space were instead finite (as in the opening example) then generically a price $V(\Pr(a|s_1, s_2))$ would reveal $\Pr(a|s_1, s_2)$. Moreover, note that conditioning on volume gives agents no extra information in the interesting part of the equilibrium, namely $\{a, b\} \times S_1^T \times S_2^T$, since here the price of p alone tells agents that the signal pair lies in $S_1^T \times S_2^T$, which is the only information communicated in equilibrium.

6. Complete markets

No-trade theorems are predicated on the assumption that agents have already exhausted all preference motives for trade (i.e., Pareto efficiency). This assumption is, in turn, often justified by appeal to an assumption of complete markets (see Milgrom and Stokey [32, p. 17]).[24] Logically separate, the extent of market completeness is related to the number of assets available for trade. In turn, the dimensionality of the price vector affects how much information is revealed in equilibrium: see Allen [1,2], or for a more specific example, Marín and Rahi [31].

By construction, our model ensures that agents have no preference motive for trade — regardless of the extent of market completeness. However, the extent to which trade reveals information is critically important for our results. On the one hand, by part (ii) of Theorem 1 the buyer must learn something from trade. On the other hand, an immediate implication of part (iii) of Theorem 1 is that a trade must stop short of revealing all information.

Earlier, we showed how the trade constructed in part (i) of Theorem 1 can be decentralized. In doing so, we assumed that the buyer and seller trade only the main asset of our model. In this section we show that the possibility of decentralized trade depends critically on the extent of market completeness, because this affects the informativeness of equilibrium prices.

Specifically, we move towards market completeness and introduce *action-securities* that pay 1 if the asset owner takes a particular pre-specified action, and 0 otherwise. These securities are in zero net supply. A GREE now allows agents to condition on the price and volume of each of the traded securities. For conciseness, we restrict attention to the case where there are just two possible actions, $X = \{A, B\}$.

In sharp contrast to our above decentralization of an equilibrium with strict gains from trade, for the case with action-securities we establish:

Proposition 2. *Suppose an action-security linked to action A is available for trade. Then in any GREE,[25] in almost all states both the buyer and seller are indifferent between trading and not trading both the main asset and the action-security.*

[23] See Kreps [29] and Dow and Gorton [13] for alternative ways to avoid equilibria with this unattractive property.

[24] See also Blume et al. [7].

[25] The only role that conditioning on volume plays in the proof of Proposition 2 is to rule out the kind of pathological equilibria identified by Kreps [29] — see earlier discussion on page 1687. With this caveat, the result covers REE as well as GREE.

The intuition behind Proposition 2 is straightforward. Trade in the action-security reveals what action the asset owner is going to take. But given this, the agent who ends up *without* the asset has at least as much information as the agent who ultimately holds the asset: the agent without the asset knows both his own signal, plus a summary statistic — namely, the action to be taken — of all information held by the other agent. In this case, prices communicate *too much* information, and trade becomes impossible.

Finally, in light of Proposition 2 it is worth stressing that action-securities of the type described can only exist under the relatively stringent condition that the action taken is observable and verifiable.

7. Discussion

In this paper we have shown that if asset payoffs are endogenously determined by the actions of agents, then trade based purely on informational differences is possible. This conclusion stands in sharp contrast to the existing literature, which takes asset values as exogenous. Trade transfers control of the asset from an agent who has received an uninformative signal to one who has received an informative signal. Even without the presence of noise traders, agents in our model would be prepared to spend resources to acquire information; and this information is subsequently partially revealed by trade.

The main empirical implication of our model is that trade affects the action taken: when the buyer acquires the asset, he (at least sometimes) takes an action that is different from the one the seller would have taken. As we discussed, this implication is consistent with both firm policy after takeovers and with different creditor behaviors in debt restructuring, though other explanations are certainly possible.

We conclude the paper with an extended discussion of two remaining issues:

7.1. Hedging as an action

In our model agents choose an action that directly affects the payoff produced by the asset. In some circumstances it is possible to reinterpret this action as a hedging decision taken after the asset is traded, which leaves asset cash flows unchanged and instead affects the utility of a risk-averse agent holding the asset.

Specifically, consider the following setting. The buyer and seller are now risk averse, with constant absolute risk aversion (CARA) utility, $u(x) = -e^{-\gamma x}$, and with the same degree of risk-aversion. Write R for the stochastic (but now exogenous) payoff eventually produced by the asset. The distribution of R differs across $\theta = a, b$. The initial wealth levels of the buyer and seller are W_1 and W_2 respectively.

After trading the main asset, both the buyer and seller can trade a second risky security and a risk free security with a third trader. We assume that this third trader is risk neutral, which allows us to assume that the returns of these two additional securities are independent of the buyer and seller's demands for them. We normalize the gross risk free return to unity, and write r for the gross return of the second risky security, which we assume is independent of θ.

Importantly, because the seller and buyer are equally risk averse, and the main asset is indivisible, the only possible motive for trading the main asset from the seller to the buyer stems from differential information.[26]

[26] Formally, see inequality (9) below.

The expected utility of an agent with wealth W who owns the main asset and believes that the probability of fundamental a is q is given by $U(W,q) \equiv \max_x E[u(W-x+R+rx)|q]$, while the expected utility of an agent who does not own the main asset is simply $\bar{U}(W) \equiv \max_x E[u(W-x+rx)]$. The analogue of the trade condition (5) is $\bar{U}(W_1+\pi(\omega)) \geqslant U(W_1, Q(\omega;\mathcal{F}_1^{\kappa,\pi}))$ together with $U(W_2-\pi(\omega), Q(\omega;\mathcal{F}_2^{\kappa,\pi})) \geqslant \bar{U}(W_2)$. Given the CARA assumption, there exists a negative-valued function v and a negative constant $\bar{v}$ such that $U(W,q)=e^{-\gamma W}v(q)$ and $\bar{U}(W)=e^{-\gamma W}\bar{v}$. Consequently, the analogue of the trade condition (5) can be written as

$$-\frac{1}{\gamma}\ln\frac{v(Q(\omega;\mathcal{F}_2^{\kappa,\pi}))}{\bar{v}} \geqslant \pi(\omega) \geqslant -\frac{1}{\gamma}\ln\frac{v(Q(\omega;\mathcal{F}_1^{\kappa,\pi}))}{\bar{v}}, \tag{9}$$

which is identical to (5) with $V(q)$ replaced by $-\frac{1}{\gamma}\ln\frac{v(q)}{\bar{v}}$. All our prior results then extend directly to this alternative interpretation.

Note that because this reinterpretation relates to an exchange economy, the no-trade theorem implies that the initial allocation is Pareto inefficient. Specifically, the initial allocation has the seller exposed to the entire risk R of the main asset, while the Pareto efficient allocation would entail allocating this risk entirely to the risk-neutral third trader. As such, this reinterpretation is applicable only when the main asset is indivisible, and has unverifiable cash flows (so that no mimicking security can exist) that only the buyer and seller can access (but that the third trader cannot, which is why he does not hold the asset).

Under this reinterpretation, our model is a simple and stylized model of dynamic trading between differentially informed agents. In a first round of trading, differentially informed agents trade in anticipation of engaging in a second round of trade. Possessing more information in the first round increases an agent's continuation utility in the second round. Relative to the large literature on dynamic trading, in our case trade in the first round takes place only for informational reasons[27]: to reiterate, absent informational differences there is no reason for the buyer to hold the indivisible main asset instead of the seller.

7.2. Information used to evaluate the trade

Kreps [29] contains an early statement of the no-trade conclusion, stated for competitive (REE) equilibria. In competitive settings, agents are effectively able to decline a trade after observing the information revealed by the terms of trade. As discussed on page 1682, we have conducted our analysis under the same assumption. However, and as forcefully observed by Dow et al. [16],[28] the no-trade result still holds if instead agents must agree (or not) to a trade based only on coarser information, such as each agent's pre-trade information. As such, no-trade theorems are not driven by information revelation, in the sense that they do not depend on an agent being able to reject a trade after seeing the proposed terms of trade.

In contrast, in our setting in which information-based trade is possible, the information that agents use to evaluate a trade does matter, and using coarser information makes trade easier. To illustrate this point, we give an example in which (i) agents evaluate a trade using just their pre-trade private information (i.e., $\mathcal{F}_1$ and $\mathcal{F}_2$), as opposed to information revealed by trade (i.e.,

[27] See, for example, Grundy and McNichols [23], Wang [42], and Foster and Viswanathan [18]. In these and other papers, trade between differentially informed investors takes place in the presence of a stochastic supply shock (i.e., noise traders).

[28] See their Proposition 2.8. See also Conclusion 2 of Rubinstein and Wolinsky [36].

$\mathcal{F}_1^{\kappa,\pi}$ and $\mathcal{F}_2^{\kappa,\pi}$), (ii) the precision condition fails, but (iii) common knowledge of strict gains from trade exist. Together with Theorem 1, this example demonstrates that the information agents use to evaluate a trade affects the circumstances under which common knowledge of gains from trade can exist. In other words, in our setting — unlike in the standard no-trade framework — it matters whether agents can commit to a trading mechanism before seeing the actual terms of trade.

The example is as follows. The two fundamentals are equally likely, $\Pr(a) = \Pr(b)$. There are two actions, $X = \{A, B\}$. Payoffs are $v(A, a) = 2$, $v(A, b) = 0$, $v(B, a) = v(B, b) = 1$. Note that the precision condition is violated because V is monotonically increasing.

Define $\underline{s}_1$ by $\Pr(a|\underline{s}_1) = 1/2$. Choose any $\bar{s}_2$. Note that $\Pr(a|s_2 < \bar{s}_2) < 1/2 < \Pr(a|s_2 > \bar{s}_2)$. Choose $\bar{s}_1 > \underline{s}_1$ such that $\Pr(a|\bar{s}_1 \text{ and } s_2 < \bar{s}_2) < 1/2$.

Consider the trade (κ, π) in which the buyer buys the asset for a price 1 if and only if $s_1 \in (\underline{s}_1, \bar{s}_1)$ and $s_2 < \bar{s}_2$. Because the ultimate asset-owner must decide between actions A and B after trade occurs, we also need to specify the post-trade information, $\mathcal{F}_1^{\kappa,\pi}$ and $\mathcal{F}_2^{\kappa,\pi}$: when the seller sees a signal $s_1 \in (\underline{s}_1, \bar{s}_1)$, he learns whether the buyer's signal is in $(-\infty, \bar{s}_2)$ or $(\bar{s}_2, \infty)$. When the buyer sees a signal $s_2 < \bar{s}_2$, he learns whether the seller's signal is in $(\underline{s}_1, \bar{s}_1)$ or $\mathbb{R} \setminus (\underline{s}_1, \bar{s}_1)$.

We will show that there is common knowledge of gains from trade when the trade is evaluated using the agents' pre-trade private information, i.e., $\mathcal{F}_1$ and $\mathcal{F}_2$. Note that we deliberately follow the no-trade literature and say nothing about what trading mechanism leads to this allocation. Instead, all we impose is the common knowledge of gains from trade requirement: agents are allowed to stick with the no-trade outcome (no transfer of asset or money) after seeing their private signals.

Because the asset is worth a minimum of 1, for all signal realizations s_2 the buyer is trivially at least weakly better off under the trade. Since no transfer occurs if $s_1 \notin (\underline{s}_1, \bar{s}_1)$, the seller is trivially indifferent between trade and no-trade for these signal realizations. Hence the only non-trivial step is to show that for all $s_1 \in (\underline{s}_1, \bar{s}_1)$,

$$1 \times \Pr(s_2 < \bar{s}_2|s_1) + V\bigl(\Pr(a|s_1, s_2 > \bar{s}_2)\bigr) \Pr(s_2 > \bar{s}_2|s_1) > V\bigl(\Pr(a|s_1)\bigr). \tag{10}$$

First note that since information is always weakly valuable,

$$\begin{aligned} &V\bigl(\Pr(a|s_1, s_2 < \bar{s}_2)\bigr) \Pr(s_2 < \bar{s}_2|s_1) + V\bigl(\Pr(a|s_1, s_2 > \bar{s}_2)\bigr) \Pr(s_2 > \bar{s}_2|s_1) \\ &\quad \geqslant V\bigl(\Pr(a|s_1)\bigr). \end{aligned} \tag{11}$$

By construction, for all $s_1 \in (\underline{s}_1, \bar{s}_1)$ an asset owner who believes the probability of a is either $\Pr(a|s_1)$ or $\Pr(a|s_1, s_2 > \bar{s}_2)$ takes action A, while an asset owner who believes the probability of a is $\Pr(a|s_1, s_2 < \bar{s}_2)$ takes action B. Consequently, giving the seller information about whether s_2 is above or below $\bar{s}_2$ is strictly valuable, since it changes his action with strictly positive probability. Moreover, since $V(\Pr(a|s_1, s_2 < \bar{s}_2)) = 1$ the left-hand sides of inequalities (10) and (11) coincide. Hence (10) holds, establishing that common knowledge of strict gains from trade exist in the example when agents evaluate the trade using their pre-trade private information — even though the precision condition does not hold.

Appendix A. Mathematical proofs

Proof of Lemma 1. Sufficiency is immediate. For necessity, suppose there is common knowledge of strict gains from trade. So there exists a set D with the properties given in Definition 1. Since $\pi(\omega) = 0$ for almost all $\omega \in D$ for which the seller keeps the assets, inequalities (3) and (4) hold with equality for almost all such $\omega \in D$. So the event $D \cap \{\omega \mid \kappa(\omega) = 2\}$ is non-null, and moreover, lies in $\mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$. This completes the proof. □

Lemma 2. *The precision condition is satisfied if and only if V is non-monotone.*

Proof. It is immediate that the precision condition implies that V is non-monotone. For the reverse implication, note that non-monotonicity of V together with convexity imply that there exist $\underline{q} \in (0,1)$ and $\bar{q} \in [\underline{q}, 1)$ such that V is strictly decreasing over $[0, \underline{q})$ and strictly increasing over $(\bar{q}, 1]$. Fix $\varepsilon < \min\{\underline{q}, 1-\bar{q}\}$. Let $\bar{h} = \max\{h(\underline{q}-\varepsilon), h(\bar{q}+\varepsilon)\}$. Then if q and q' are such that $h(q) > h(q') \geqslant \bar{h}$ and $|q-q'| < \varepsilon$, either $q < q' \leqslant \underline{q}$ or $q > q' \geqslant \bar{q}$. In either case $V(q) > V(q')$ and hence the precision condition is satisfied. □

Proof of Theorem 1.

Part (i), necessity. Suppose to the contrary that the precision condition is not satisfied but there is a trade (κ, π) such that common knowledge of strict gains from trade exists. By Lemma 2, V is monotone. Without loss, suppose V is weakly increasing.

By Lemma 1, there exist $D \in \mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$ and $D' \in \mathcal{F}$ satisfying the properties stated in the lemma. It follows from (5) that $Q(\omega; \mathcal{F}_2^{\kappa,\pi}) \geqslant Q(\omega; \mathcal{F}_1^{\kappa,\pi})$ for almost all $\omega \in D$ and $Q(\omega; \mathcal{F}_2^{\kappa,\pi}) > Q(\omega; \mathcal{F}_1^{\kappa,\pi})$ for almost all $\omega \in D'$. By the definition of conditional probability, for $i = 1, 2$,

$$\int_D Q\big(\omega; \mathcal{F}_i^{\kappa,\pi}\big)\,\mu(d\omega) = \mu\big(D \cap \{a\} \times \mathbb{R}^2\big),$$

and so

$$\int_D \big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big) - Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big)\,\mu(d\omega) = 0. \tag{12}$$

This gives a contradiction, since the left-hand side of (12) can be decomposed as

$$\int_{D\setminus D'} \big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big) - Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big)\,\mu(d\omega) + \int_{D'} \big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big) - Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\big)\,\mu(d\omega),$$

where the first term is weakly positive and the second term is strictly positive.

Part (i), sufficiency. We prove the sufficiency of the precision condition for common knowledge of strict gains from trade by showing that an equilibrium of the form described on page 1684 exists.

We start with some preliminaries. Recall that $L_i(s_i)$ denotes the likelihood ratio of signal s_i; likewise, for any set S such that $\eta_i^a(S) > 0$, we let $L_i(S)$ denote the likelihood ratio $\eta_i^a(S)/\eta_i^b(S)$. The asset value V is defined as a function of q, the probability the asset holder attaches to fundamental a. In the trade equilibria under consideration, for the seller the conditional probability q is of the form $\Pr(a|s_1, s_2 \notin S_2^T)$, while for the buyer it is of the form $\Pr(a|s_1 \in S_1^T, s_2)$. It is convenient to rewrite these probabilities as

$$\Pr\big(a\big|s_1, s_2 \notin S_2^T\big) = \frac{\frac{\Pr(a)}{\Pr(b)} L_1(s_1) L_2(\mathbb{R} \setminus S_2^T)}{\frac{\Pr(a)}{\Pr(b)} L_1(s_1) L_2(\mathbb{R} \setminus S_2^T) + 1},$$

$$\Pr\big(a\big|s_1 \in S_1^T, s_2\big) = \frac{\frac{\Pr(a)}{\Pr(b)} L_1(S_1^T) L_2(s_2)}{\frac{\Pr(a)}{\Pr(b)} L_1(S_1^T) L_2(s_2) + 1}.$$

Next, define a mapping from likelihood ratios to probabilities by

$$q(L) \equiv \frac{\frac{\Pr(a)}{\Pr(b)}L}{\frac{\Pr(a)}{\Pr(b)}L + 1} \quad \text{for any } L \in [0, \infty),$$

along with a transformation V^ℓ of V that takes a likelihood ratio L as its argument, i.e., $V^\ell \equiv V \circ q$. By hypothesis the precision condition is satisfied, and so V is non-monotonic. Combined with convexity of V, this implies that there exist probabilities $q^* \in (0, 1)$ and $q^{**} \in [q^*, 1)$ such that V is strictly decreasing over $[0, q^*)$, flat over $[q^*, q^{**}]$, and strictly increasing over $(q^{**}, 1]$. Since q is strictly increasing in L, $L^* = q^{-1}(q^*)$ and $L^{**} = q^{-1}(q^{**})$ are well defined, and V^ℓ is strictly decreasing over $[0, L^*)$, flat over $[L^*, L^{**}]$, and strictly increasing over (L^{**}, ∞).

We show there exists an equilibrium of the type described, i.e., $S_1^T \equiv [\underline{s}_1, \bar{s}_1]$ and $S_2^T \equiv \mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)$. If the seller sees signal s_1 his payoff from offering to sell is

$$\Pr\left(s_2 \in S_2^T \middle| s_1\right) p + \Pr\left(s_2 \notin S_2^T \middle| s_1\right) V^\ell\left(L_1(s_1) L_2\left(\mathbb{R} \setminus S_2^T\right)\right),$$

while his payoff from not offering to sell is

$$\Pr\left(s_2 \in S_2^T \middle| s_1\right) V^\ell\left(L_1(s_1) L_2\left(S_2^T\right)\right) + \Pr\left(s_2 \notin S_2^T \middle| s_1\right) V^\ell\left(L_1(s_1) L_2\left(\mathbb{R} \setminus S_2^T\right)\right).$$

Thus it is a best response for the seller to offer to sell whenever $s_1 \in S_1^T$ if and only if

$$V^\ell\left(L_1(s_1) L_2\left(S_2^T\right)\right) \leqslant p \quad \text{for all } s_1 \in S_1^T,$$
$$V^\ell\left(L_1(s_1) L_2\left(S_2^T\right)\right) \geqslant p \quad \text{for all } s_1 \notin S_1^T.$$

By continuity and the shape of V^ℓ, these conditions are satisfied if and only if

$$V^\ell\left(L_1(\underline{s}_1) L_2\left(S_2^T\right)\right) = V^\ell\left(L_1(\bar{s}_1) L_2\left(S_2^T\right)\right) = p. \tag{13}$$

Likewise, in order for the buyer to offer to buy whenever $s_2 \in S_2^T$,

$$V^\ell\left(L_1\left(S_1^T\right) L_2(s_2)\right) \geqslant p \quad \text{for all } s_2 \in S_2^T,$$
$$V^\ell\left(L_1\left(S_1^T\right) L_2(s_2)\right) \leqslant p \quad \text{for all } s_2 \notin S_2^T,$$

and these conditions are satisfied if and only if

$$V^\ell\left(L_1\left(S_1^T\right) L_2(\underline{s}_2)\right) = V^\ell\left(L_1\left(S_1^T\right) L_2(\bar{s}_2)\right) = p. \tag{14}$$

Thus a trade equilibrium exists if and only if there exist $\underline{s}_1, \bar{s}_1 \neq \underline{s}_1, \underline{s}_2, \bar{s}_2 \neq \underline{s}_2$ such that (13) and (14) hold. From the shape of V^ℓ, there exists a unique pair $\underline{L}$ and $\bar{L}$ such that $\underline{L} < L^* \leqslant L^{**} < \bar{L}$, and $V^\ell(\underline{L}) = V^\ell(\bar{L}) = p$. Consequently, a trade equilibrium of the type described exists if and only if there exist $\underline{s}_1, \bar{s}_1 \neq \underline{s}_1, \underline{s}_2, \bar{s}_2 \neq \underline{s}_2$ satisfying the following system of four equations:

$$L_1(\underline{s}_1) L_2\left(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)\right) = \underline{L}, \tag{15}$$
$$L_1(\bar{s}_1) L_2\left(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)\right) = \bar{L}, \tag{16}$$
$$L_1\left([\underline{s}_1, \bar{s}_1]\right) L_2(\underline{s}_2) = \underline{L}, \tag{17}$$
$$L_1\left([\underline{s}_1, \bar{s}_1]\right) L_2(\bar{s}_2) = \bar{L}. \tag{18}$$

To complete the proof we show that such a quadruple does exist. First note that (15) and (16) imply

$$\frac{L_1(\underline{s}_1)}{L_1(\bar{s}_1)} = \frac{\underline{L}}{\bar{L}} < 1 \tag{19}$$

and (17) and (18) imply

$$\frac{L_2(\underline{s}_2)}{L_2(\bar{s}_2)} = \frac{\underline{L}}{\bar{L}} < 1. \tag{20}$$

Fix $\underline{s}_1$ and solve for $\bar{s}_1(\underline{s}_1) > \underline{s}_1$ from (19). Similarly solve for $\bar{s}_2(\underline{s}_2) > \underline{s}_2$ from (20). Substituting for $\bar{s}_1(\underline{s}_1)$ and $\bar{s}_2(\underline{s}_2)$, rewrite (15) and (17) as

$$L_1(\underline{s}_1)L_2\big(\mathbb{R} \setminus \big(\underline{s}_2, \bar{s}_2(\underline{s}_2)\big)\big) = \underline{L}, \tag{21}$$

$$L_1\big(\big[\underline{s}_1, \bar{s}_1(\underline{s}_1)\big]\big)L_2(\underline{s}_2) = \underline{L}. \tag{22}$$

Observe that $\bar{s}_1(\underline{s}_1) \to \pm\infty$ as $\underline{s}_1 \to \pm\infty$. Consequently $L_1([\underline{s}_1, \bar{s}_1(\underline{s}_1)]) \to \infty$ as $\underline{s}_1 \to \infty$ and $L_1([\underline{s}_1, \bar{s}_1(\underline{s}_1)]) \to 0$ as $\underline{s}_1 \to -\infty$. Thus from (22) define $\underline{s}_2(\underline{s}_1)$, and note that $\underline{s}_2(\underline{s}_1) \to \mp\infty$ as $\underline{s}_1 \to \pm\infty$.

Also observe that $\bar{s}_2(\underline{s}_2) \to \pm\infty$ as $\underline{s}_2 \to \pm\infty$, and so $L_2(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)) \to 1$ as $\underline{s}_2 \to \pm\infty$. So substituting in for $\underline{s}_2(\underline{s}_1)$, the left-hand side of (21) approaches 0 as $\underline{s}_1 \to -\infty$ and grows without bound as $\underline{s}_1 \to \infty$. By continuity it follows that there exists some $\underline{s}_1$ such that

$$L_1(\underline{s}_1)L_2\big(\mathbb{R} \setminus \big(\underline{s}_2(\underline{s}_1), \bar{s}_2\big(\underline{s}_2(\underline{s}_1)\big)\big)\big) = \underline{L},$$

completing the proof.

Part (ii). Suppose to the contrary that there is a trade (κ, π) such that common knowledge of strict gains from trade exists, and such that the buyer learns nothing whenever he acquires the asset. By Lemma 1, there exist $D \in \mathcal{F}_1^{\kappa,\pi} \cap \mathcal{F}_2^{\kappa,\pi}$ and $D' \subset D$ satisfying the properties stated in the lemma.

Choose an integer n such that $D' \cap \{a, b\} \times \mathbb{R} \times [n, n+1]$ has strictly positive mass. Let D_n (respectively, D'_n) be the subset of D such that the buyer's signal lies in $[n, n+1]$, and (5) holds (respectively, (5) holds with at least one inequality strict).[29] Since the buyer learns nothing when he acquires the asset, $Q((\theta, s_1, s_2); \mathcal{F}_2^{\kappa,\pi}) = Q((\theta, s_1, s_2); \mathcal{F}_2) = \Pr(a|s_2)$ for all $(\theta, s_1, s_2) \in D$. Let $\underline{q} = \Pr(a|s_2 = n)$ and $\bar{q} = \Pr(a|s_2 = n+1)$, so that

$$Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big) \in [\underline{q}, \bar{q}] \quad \text{for } \omega \in D_n. \tag{23}$$

We claim that

$$\inf_{\omega \in D_n} Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big) < \underline{q} < \bar{q} < \sup_{\omega \in D_n} Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big). \tag{24}$$

This implies the result, as follows. For $\omega \in D_n$, the buyer pays less than his valuation of the asset, i.e., (5) holds. Combined with (23) and the convexity of V, it follows that

$$\pi(\omega) \leqslant V\big(Q\big(\omega; \mathcal{F}_2^{\kappa,\pi}\big)\big) \leqslant \max\big\{V(\underline{q}), V(\bar{q})\big\}.$$

Given that V is convex, (24) implies

$$\max\big\{V(\underline{q}), V(\bar{q})\big\} \leqslant \max\Big\{V\Big(\inf_{\omega \in D_n} Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\Big), V\Big(\sup_{\omega \in D_n} Q\big(\omega; \mathcal{F}_1^{\kappa,\pi}\big)\Big)\Big\}. \tag{25}$$

On the one hand, consider the case in which inequality (25) holds strictly. Then since V is continuous there must exist $\omega \in D_n$ such that $V(Q(\omega; \mathcal{F}_1^{\kappa,\pi})) > \pi(\omega)$, i.e., the seller's valuation strictly exceeds the price paid, contradicting inequality (5).

[29] Note that D_n and D'_n are both $\mathcal{F}$-measurable since both inequalities in (5) contain only $\mathcal{F}$-measurable functions.

On the other hand, consider the case in which inequality (25) holds with equality. Given the convexity of V, it follows that V must be constant over the interval between $\inf_{\omega\in D_n} Q(\omega;\mathcal{F}_1^{\kappa,\pi})$ and $\sup_{\omega\in D_n} Q(\omega;\mathcal{F}_1^{\kappa,\pi})$. But then the buyer and seller value the asset equally over all D_n, contradicting the supposition of strict gains from trade over the non-null subset D_n' of D_n. This completes the proof of the claim.

To complete the proof of part (ii), we must establish (24). First, suppose that contrary to the claim, $\inf_{\omega\in D_n} Q(\omega;\mathcal{F}_1^{\kappa,\pi}) \geqslant \underline{q}$. By definition, $D\in\mathcal{F}_1^{\kappa,\pi}$, so from the definition of conditional probability, for any s_1

$$\int\limits_{D\cap(\{a,b\}\times(-\infty,s_1)\times\mathbb{R})} Q\big(\omega;\mathcal{F}_1^{\kappa,\pi}\big)\,\mu(d\omega)=\mu\big(D\cap\big(\{a\}\times(-\infty,s_1)\times\mathbb{R}\big)\big).$$

Since $D_n\subset D$ and by supposition $Q(\omega;\mathcal{F}_1^{\kappa,\pi})\geqslant\underline{q}$ over D_n,

$$\begin{aligned}\int\limits_{D\cap(\{a,b\}\times(-\infty,s_1)\times\mathbb{R})} Q\big(\omega;\mathcal{F}_1^{\kappa,\pi}\big)\,\mu(d\omega) &\geqslant \int\limits_{D_n\cap(\{a,b\}\times(-\infty,s_1)\times\mathbb{R})} Q\big(\omega;\mathcal{F}_1^{\kappa,\pi}\big)\,\mu(d\omega)\\ &\geqslant \underline{q}\mu\big(D_n\cap\big(\{a,b\}\times(-\infty,s_1)\times\mathbb{R}\big)\big).\end{aligned}$$

So

$$\underline{q}\leqslant\frac{\mu(D\cap(\{a\}\times(-\infty,s_1)\times\mathbb{R}))}{\mu(D_n\cap(\{a,b\}\times(-\infty,s_1)\times\mathbb{R}))}. \tag{26}$$

By definition, $D\in\mathcal{F}_2^{\kappa,\pi}$ and the buyer acquires the asset in all states in D. Since the buyer learns nothing when he acquires the asset, $D\in\mathcal{F}_2$ and so is of the form $\{a,b\}\times\mathbb{R}\times S_2^T$, where $S_2^T\in\mathcal{B}$. Since D and D_n are non-null, both $\eta_1^\theta(S_2^T)$ and $\eta_1^\theta(S_2^T\cap[n,n+1])$ are strictly positive for $\theta=a,b$. So inequality (26) rewrites to

$$\underline{q}\leqslant\frac{\Pr(a)F_1^a(s_1)\eta_1^a(S_2^T)}{\Pr(a)F_1^a(s_1)\eta_1^a(S_2^T\cap[n,n+1])+\Pr(b)F_1^b(s_1)\eta_1^b(S_2^T\cap[n,n+1])}.$$

But since the likelihood ratio is unbounded (see (1)) the right-hand side converges to 0 as $s_1\to-\infty$,[30] giving a contradiction and thus showing $\inf_{\omega\in D_n} Q(\omega;\mathcal{F}_1^{\kappa,\pi})<\underline{q}$. A parallel argument implies $\bar{q}<\sup_{\omega\in D_n} Q(\omega;\mathcal{F}_1^{\kappa,\pi})$, completing the proof.

Part (iii). Suppose to the contrary that there is a trade (κ,π) such that common knowledge of strict gains from trade exists, but the buyer almost always takes the same action the seller would take if he controlled the asset in the same state. So $V(Q(\omega;\mathcal{F}_2^{\kappa,\pi}))=V(Q(\omega;\mathcal{F}_1^{\kappa,\pi}))$ for almost all $\omega\in\Omega$ such that $\kappa(\omega)=2$. But this contradicts (6). □

Proof of Proposition 1. The asset value V is convex. The key to the proof is the following observation: for any set $S_2\subset\mathbb{R}$, Jensen's inequality implies

[30] As noted in footnote 14, the assumption of conditional independence of the seller and buyer signals is used in this argument. More generally, the right-hand side of (26) can be written as $\frac{\mu(D_n\cap(\{a\}\times(-\infty,s_1)\times\mathbb{R}))}{\mu(D_n\cap(\{a,b\}\times(-\infty,s_1)\times\mathbb{R}))}\frac{\mu(D\cap(\{a\}\times(-\infty,s_1)\times\mathbb{R}))}{\mu(D_n\cap(\{a\}\times(-\infty,s_1)\times\mathbb{R}))}$. Under the relatively mild assumptions that the first of these fractions converges to 0 as $s_1\to-\infty$, and the second is bounded above as $s_1\to-\infty$, the assumption of conditional independence can be straightforwardly relaxed here.

$$E\big[V\big(\Pr\big(a\big|\{s_1\}\times S_2\big)\big)\big|s_2\in S_2\big]$$
$$=\sum_{\theta=a,b}\Pr(\theta|s_2\in S_2)\int_{-\infty}^{\infty}V\big(\Pr\big(a\big|\{s_1\}\times S_2\big)\big)f_1^{\theta}(s_1)\,ds_1$$
$$\geqslant\sum_{\theta=a,b}\Pr(\theta|s_2\in S_2)V\big(\Pr(a|S_2)\big)=V\big(\Pr(a|S_2)\big).$$

Given the precision condition V cannot be linear over its domain [0, 1]. The unbounded likelihood assumption ensures that $\Pr(a|\{s_1\}\times S_2)$ varies from 0 to 1 as s_1 increases from $-\infty$ to ∞. Hence the above inequality must be strict. Likewise, and by a parallel argument, for any $S_1\subset\mathbb{R}$, $E[V(\Pr(a|S_1\times\{s_2\}))|s_1\in S_1]>V(\Pr(a|S_1))$.

We show that the equilibrium established in part (i) of Theorem 1 remains an equilibrium when the buyer and seller must pay k to acquire their signals. For an information acquisition cost of $k=0$, the seller's equilibrium utility is

$$\begin{aligned}&p\Pr\big(s_1\in S_1^T,s_2\in S_2^T\big)\\&\quad+E\big[V\big(\Pr\big(a\big|\{s_1\}\times S_2^T\big)\big)\big|s_1\notin S_1^T,s_2\in S_2^T\big]\Pr\big(s_1\notin S_1^T,s_2\in S_2^T\big)\\&\quad+E\big[V\big(\Pr\big(a\big|\{s_1\}\times\mathbb{R}\setminus S_2^T\big)\big)\big]\Pr\big(s_2\notin S_2^T\big).\end{aligned}\tag{27}$$

Because the seller could instead always offer to sell, this quantity exceeds

$$p\Pr\big(s_2\in S_2^T\big)+E\big[V\big(\Pr\big(a\big|\{s_1\}\times\mathbb{R}\setminus S_2^T\big)\big)\big|s_2\notin S_2^T\big]\Pr\big(s_2\notin S_2^T\big),$$

which by the above is in turn strictly greater than

$$p\Pr\big(s_2\in S_2^T\big)+V\big(\Pr\big(a\big|\mathbb{R}\setminus S_2^T\big)\big)\Pr\big(s_2\notin S_2^T\big).$$

This last expression equals the seller's payoff under the deviation in which he does not buy his signal and always trades. Similarly, the seller's equilibrium utility (27) is greater than his payoff from never trading,

$$\begin{aligned}&E\big[V\big(\Pr\big(a\big|\{s_1\}\times S_2^T\big)\big)\big|s_2\in S_2^T\big]\Pr\big(s_2\in S_2^T\big)\\&\quad+E\big[V\big(\Pr\big(a\big|\{s_1\}\times\mathbb{R}\setminus S_2^T\big)\big)\big|s_2\notin S_2^T\big]\Pr\big(s_2\notin S_2^T\big),\end{aligned}$$

which is in turn strictly greater than

$$V\big(\Pr\big(a\big|S_2^T\big)\big)\Pr\big(s_2\in S_2^T\big)+V\big(\Pr\big(a\big|\mathbb{R}\setminus S_2^T\big)\big)\Pr\big(s_2\notin S_2^T\big),$$

the value of the asset to the seller if he observes only the buyer's announcement of whether or not he is prepared to buy. So for all information acquisition costs k that are sufficiently low the seller chooses to buy his information.

For an information acquisition cost of $k=0$, the buyer's equilibrium utility is

$$E\big[V\big(\Pr\big(a\big|S_1^T\times\{s_2\}\big)\big)-p\big|s_1\in S_1^T,s_2\in S_2^T\big]\Pr\big(s_1\in S_1^T,s_2\in S_2^T\big).$$

Because the buyer could instead always offer to buy, this exceeds

$$E\big[V\big(\Pr\big(a\big|S_1^T\times\{s_2\}\big)\big)-p\big|s_1\in S_1^T\big]\Pr\big(S_1^T\big),$$

which in turn strictly exceeds

$$\big(V\big(\Pr\big(a\big|S_1^T\big)\big)-p\big)\Pr\big(S_1^T\big),$$

the buyer's payoff under the deviation in which he does not buy his signal and always trades. Finally, if the buyer deviates to not buying the signal and never trading, his payoff is simply zero, and which is strictly less than his equilibrium utility. Again, for all information acquisition costs k that are sufficiently low the buyer chooses to buy his information. □

Proof of Proposition 2. Fix a GREE, with $p : \Omega \to \mathbb{R}$ the equilibrium price of the main asset, and Ω^T the subset of states in which the buyer acquires the main asset. Partition Ω^T into Ω_A^T and Ω_B^T, according to the action the buyer takes. Denote agent i's equilibrium information by $\mathcal{F}_i^{post}$. Let $Q_A(\omega)$ be the seller's conditional probability that the state is in Ω_A^T in state ω.

The heart of the proof is the following claim, proved below:

Claim. $Q_A(\omega) = 1$ *for almost all* $\omega \in \Omega_A^T$, *and* $Q_A(\omega) = 0$ *for almost all* $\omega \in \Omega_B^T$.

The claim establishes that the seller knows what action the buyer will take in every state in which he acquires the asset. Consider now the counterfactual in which the seller keeps the asset over Ω^T, but continues to have information $\mathcal{F}_1^{post}$. By the claim, it is feasible for the seller to exactly mimic the action taken by the buyer in the conjectured equilibrium. Consequently, the seller's valuation of the asset over Ω^T is at least as high as the buyer's,

$$\int_{\Omega^T} V\big(Q\big(\omega; \mathcal{F}_1^{post}\big)\big)\,\mu(d\omega) \geqslant \int_{\Omega^T} V\big(Q\big(\omega; \mathcal{F}_2^{post}\big)\big)\,\mu(d\omega).$$

But the equilibrium conditions imply that for almost all $\omega \in \Omega^T$,

$$V\big(Q\big(\omega; \mathcal{F}_2^{post}\big)\big) \geqslant p(\omega) \geqslant V\big(Q\big(\omega; \mathcal{F}_1^{post}\big)\big).$$

These two inequalities together imply that for almost all $\omega \in \Omega^T$,

$$V\big(Q\big(\omega; \mathcal{F}_2^{post}\big)\big) = p(\omega) = V\big(Q\big(\omega; \mathcal{F}_1^{post}\big)\big),$$

so that both the buyer and seller are indifferent between trading the main asset and not trading it. The claim also implies that the equilibrium price of the action-security must equal 1 in almost all $\omega \in \Omega_A^T$ and 0 in almost all $\omega \in \Omega_B^T$, likewise implying that both the buyer and seller are indifferent between trading the action-security.

Finally, an analogous argument to the above implies that over the remaining states $\Omega \setminus \Omega^T$ both the buyer and seller are indifferent between trading and not trading both the main asset and the action security.

Proof of Claim. We prove that $Q_A(\omega) = 1$ for almost all $\omega \in \Omega_A^T$ (the proof of the other statement is identical).

The case $\mu(\Omega_A^T) = 0$ is immediate.

For the case $\mu(\Omega_A^T) > 0$, we first show that $Q_A(\omega) \in \{0, 1\}$ for almost all $\omega \in \Omega_A^T$. Suppose to the contrary that there exists $\tilde{\Omega} \subset \Omega_A^T$ such that $\mu(\tilde{\Omega}) > 0$ and $Q_A(\omega) \in (0, 1)$ for all $\omega \in \tilde{\Omega}$. Let $\check{\Omega}$ be the set of all states $\omega \in \Omega^T$ such that there exists some $\tilde{\omega} \in \tilde{\Omega}$ at which the seller places the same probability on action A, i.e., $Q_A(\omega) = Q_A(\tilde{\omega})$, and the price-volume vectors in ω and $\tilde{\omega}$ coincide. The set $\check{\Omega}$ is $\mathcal{F}_1^{post}$-measurable, and contains $\tilde{\Omega}$. Because $Q_A(\omega) < 1$ for all $\omega \in \check{\Omega}$, we know

$$\int_{\check{\Omega}} Q_A(\omega)\,\mu(d\omega) < \mu(\check{\Omega}).$$

By the definition of conditional probabilities,

$$\int_{\check{\Omega}} Q_A(\omega)\,\mu(d\omega) = \mu\big(\check{\Omega} \cap \Omega_A^T\big).$$

So $\mu(\check{\Omega} \cap \Omega_A^T) < \mu(\check{\Omega})$, implying that $\check{\Omega} \cap \Omega_B^T$ is non-null. Let ω_B be a typical member of $\check{\Omega} \cap \Omega_B^T$. By the construction of $\check{\Omega}$, there exists $\omega_A \in \Omega_A^T$ such that $Q_A(\omega_A) = Q_A(\omega_B) \in (0,1)$ and the price-volume vectors coincide at ω_A and ω_B.

The common price of the action-security in ω_A and ω_B cannot be 1: if it were, both the buyer and seller strictly desire to sell the action-security in state ω_B, and so the action-security market fails to clear.

The common price of the action-security in ω_A and ω_B cannot be 0: if it were, both the buyer and seller strictly desire to buy the action-security in state ω_A, and so the action-security market fails to clear.

The common price of the action-security in ω_A and ω_B cannot lie in $(0,1)$: if it did, the buyer would strictly desire to buy the action-security in ω_A and sell in ω_B, and so volume cannot be the same in the two states.

Since this exhausts the possibilities, we have a contradiction, and hence have shown that $Q_A(\omega) \in \{0,1\}$ for almost all $\omega \in \Omega_A^T$. Finally, since $\int_{\Omega_A} Q_A(\omega)\,\mu(d\omega) = \mu(\Omega_A)$, it follows that $Q_A(\omega) = 1$ for almost all $\omega \in \Omega_A^T$. This completes the proof of the claim.

Appendix B. Analysis of case in which buyer proposes a price

In the main text, we use a simple third-party posted price mechanism to establish the sufficiency of the precision condition for trade (Theorem 1). In this appendix, we analyze trade in the buyer-posted price mechanism described on page 1686. As noted in footnote 22, to avoid technical complications associated with infinite action sets we assume that the buyer's offer p is constrained to lie within some finite set P. We assume that P contains at least one element lying between $\min V$ and $\min\{V(0), V(1)\}$; and moreover that $0 \in P$, so that the buyer can effectively abstain from making an offer by offering the zero price.

Our first result shows that an equilibrium with meaningful trade is necessarily more complicated when the buyer chooses the price than when the price is simply imposed non-strategically. To see this, start by noting that in the equilibrium of Theorem 1 both the buyer and the seller make zero profit at the boundaries of their trade sets S_1^T and S_2^T. This is a consequence of continuity: for example, for the buyer, when $s_2 \in S_2^T$ the buyer's valuation of the asset exceeds p, while when $s_2 \notin S_2^T$ the price p exceeds the buyer's valuation.

Because the buyer has zero profits at the boundary signals $\underline{s}_2$ and $\bar{s}_2$ of S_2^T, he faces a strong temptation to offer a lower price after observing these signals. In fact, regardless of the seller's out-of-equilibrium beliefs the buyer could make strictly positive profits at at least one of $\underline{s}_2$ and $\bar{s}_2$ by offering $\tilde{p}$ between p and $\min V$. The reason is that the seller's response to $\tilde{p}$ either increases or decreases the buyer's belief that the fundamental is a relative to his equilibrium belief, and given the convex and non-monotone shape of V (from Theorem 1, the precision condition must hold) this raises the buyer's valuation at one of $\underline{s}_2$ and $\bar{s}_2$.

It follows that the only possibility for an equilibrium with trade at just one price entails trade at the lowest price in P that still exceeds $\min V$, i.e.,

$$p^* \equiv \min\{p \in P:\ p > \min V\}.$$

However, the seller only accepts this low price when he sees a signal that places his valuation between $\min V$ and p^*. When p^* is close to $\min V$ this can occur only rarely. Formally:

Proposition 3. *Suppose the precision condition holds, V is never flat,*[31] *and the buyer-posted price mechanism is used. Suppose an equilibrium exists in which trade occurs at only one price. Then* (I) *the trade price is p^*, and* (II) *the probability of trade approaches zero as $p^* \to \min V$.*

Proposition 3 implies that when the buyer chooses the price trade can occur with a meaningful probability only if the buyer makes different offers after different signals, and the seller accepts multiple offers with different probabilities. Characterizing such an equilibrium is challenging. Formally, our environment is close to a bargaining game with interdependent values and two-sided asymmetric information,[32] and we are not aware of any paper to consider such a game with more than two types.[33]

In order to illustrate the possibilities for trade when the buyer proposes the price, we focus on the simplest environment in which strategic offers by the buyer are possible, namely that in which the precision condition is satisfied — so V is non-monotone — and the offer set is $P = \{p^C, p^D, 0\}$, where $\min\{V(0), V(1)\} > p^C > p^D > \min V$. As we explain below, even here the fact that there are a continuum of buyer "types" necessitates a numerical simulation in order to verify the incentive constraints.

A trade equilibrium with trade at both prices p^C and p^D is characterized by signal sets S_2^C and S_2^D in which the buyer offers the prices p^C and p^D respectively, with corresponding signal sets S_1^C and S_1^D in which the seller accepts these offers. Given that V is convex and non-monotone the seller's acceptance sets S_1^C and S_1^D must be of the form $(\underline{s}_1^C, \bar{s}_1^C)$ and $(\underline{s}_1^D, \bar{s}_1^D)$. As in the equilibrium of Theorem 1, the seller has zero profits at the boundaries of S_1^C and S_1^D:

$$V\big(\Pr\big(a \big| \underline{s}_1^j, s_2 \in S_2^j\big)\big) = V\big(\Pr\big(a \big| \bar{s}_1^j, s_2 \in S_2^j\big)\big) = p^j \quad \text{for } j = C, D. \tag{28}$$

Since both p^C and p^D exceed $\min V$, there must exist a non-empty subset of signals at which the buyer prefers to make no offer. Again by the convexity and non-monotonicity of V, this no-offer set is an interval of the form $[\underline{s}_2^D, \bar{s}_2^D]$. Thus the equilibrium conditions for the buyer are that for $s_2 \in S_2^C$

$$\begin{aligned}&\Pr\big(s_1 \in S_1^C \big| s_2\big)\big(V\big(\Pr\big(a \big| s_1 \in S_1^C, s_2\big)\big) - p^C\big) \\ &\quad \geqslant \max\big\{0, \Pr\big(s_1 \in S_1^D \big| s_2\big)\big(V\big(\Pr\big(a \big| s_1 \in S_1^D, s_2\big)\big) - p^D\big)\big\};\end{aligned} \tag{29}$$

while for $s_2 \in S_2^D$,

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[31] The assumption that V is never flat means that a small change in precision $h(q)$ is always associated with a change in asset value $V(q)$.

[32] However, our environment is *not* a special case of such a bargaining game because the value of the asset is endogenous.

[33] Schweizer [38] analyzes the case of two types. In general, the literature on bargaining with interdependent values is small and focuses on the case of one-sided asymmetric information: see Evans [17], Vincent [41], Deneckere and Liang [11], Dal Bo and Powell [10].

$$\Pr\bigl(s_1 \in S_1^D \big| s_2\bigr)\bigl(V\bigl(\Pr\bigl(a \big| s_1 \in S_1^D, s_2\bigr)\bigr) - p^D\bigr)$$
$$\geqslant \max\bigl\{0, \Pr\bigl(s_1 \in S_1^C \big| s_2\bigr)\bigl(V\bigl(\Pr\bigl(a \big| s_1 \in S_1^C, s_2\bigr)\bigr) - p^C\bigr)\bigr\}; \tag{30}$$

and if $s_2 \in [\underline{s}_2^D, \bar{s}_2^D]$,

$$0 \geqslant \max\bigl\{V\bigl(\Pr\bigl(a \big| s_1 \in S_1^C, s_2\bigr)\bigr) - p^C, V\bigl(\Pr\bigl(a \big| s_1 \in S_1^D, s_2\bigr)\bigr) - p^D\bigr\}.$$

Finally, in light of the form the equilibrium takes in the third-party mechanism, along with the non-monotonicity of V, a natural conjecture for the form of the buyer's trade sets S_2^C and S_2^D is as follows. The buyer makes the higher offer p^C only when his signal is relatively informative, that is, S_2^C is of the form $\mathbb{R} \setminus [\underline{s}_2^C, \bar{s}_2^C]$. The buyer then makes the lower offer p^D after the remaining signals $\mathbb{R} \setminus (S_2^C \cup [\underline{s}_2^D, \bar{s}_2^D])$, so that $S_2^D = [\underline{s}_2^C, \underline{s}_2^D) \cup (\bar{s}_2^D, \bar{s}_2^C]$. By continuity, the following equalities are then *necessary* for an equilibrium of this type to exist:

$$V\bigl(\Pr\bigl(a \big| s_1 \in S_1^D, s_2\bigr)\bigr) = p^D \tag{31}$$

at $s_2 = \underline{s}_2^D, \bar{s}_2^D$; while at $s_2 = \underline{s}_2^C, \bar{s}_2^C$,

$$\Pr\bigl(s_1 \in S_1^C \big| s_2\bigr)\bigl(V\bigl(\Pr\bigl(a \big| s_1 \in S_1^C, s_2\bigr)\bigr) - p^C\bigr)$$
$$= \Pr\bigl(s_1 \in S_1^D \big| s_2\bigr)\bigl(V\bigl(\Pr\bigl(a \big| s_1 \in S_1^D, s_2\bigr)\bigr) - p^D\bigr). \tag{32}$$

Together, Eqs. (28), (31) and (32) constitute eight equations in the eight parameters $\{\underline{s}_1^C, \bar{s}_1^C, \underline{s}_1^D, \bar{s}_1^D, \underline{s}_2^C, \bar{s}_2^C, \underline{s}_2^D, \bar{s}_2^D\}$ that characterize the equilibrium of the type described.

Fig. 2 displays an example of such an equilibrium for a specific set of parameter values.[34] The figure plots the buyer's expected profits from each of the offers p^C and p^D as a function of his signal s_2. The vertical lines are drawn at the boundaries of the sets S_2^C and S_2^D, i.e., at $\underline{s}_2^C, \underline{s}_2^D, \bar{s}_2^C, \bar{s}_2^D$ respectively. The figure makes clear that the solution to Eqs. (28), (31) and (32) defines an equilibrium in this case: whenever $s_2 \in S_2^C$ the buyer prefers offering p^C to p^D, and whenever $s_2 \in S_2^D$ the buyer prefers offering p^D to p^C. The advantage of the higher offer p^C is that it is accepted more often; the disadvantage is, of course, that the buyer pays more.

The significance of this example relative to Proposition 3 is that it shows that trade can occur with significant probability when the buyer proposes different prices after different signals. That is, although the probability of trade at the lower price p^D is relatively low, the buyer sometimes offers the higher price p^C, and the seller's acceptance probability at this price is higher. Moreover, our focus on an equilibrium with trade at two prices is solely for tractability, and we conjecture that with a finer offer set P there exist equilibria with trade at a large number of different prices. We leave the further exploration of this bargaining framework for future research.

B.1. Proof of Proposition 3

From Theorem 1, the precision condition must hold. Let the likelihood ratios L_i and function V^ℓ be as defined in the proof of the sufficiency half of part (i), Theorem 1. From the same part of the proof of Theorem 1, we know that there exists L^* such that V^ℓ is strictly decreasing over $[0, L^*)$ and strictly increasing over (L^*, ∞).

[34] Unfortunately, we have not been able to characterize the conditions under which Eqs. (28), (31) and (32) have a solution; and even if we were to do so, one would still need to check that the equilibrium conditions (29) and (30) hold away from the boundaries of the S_2^C and S_2^D.

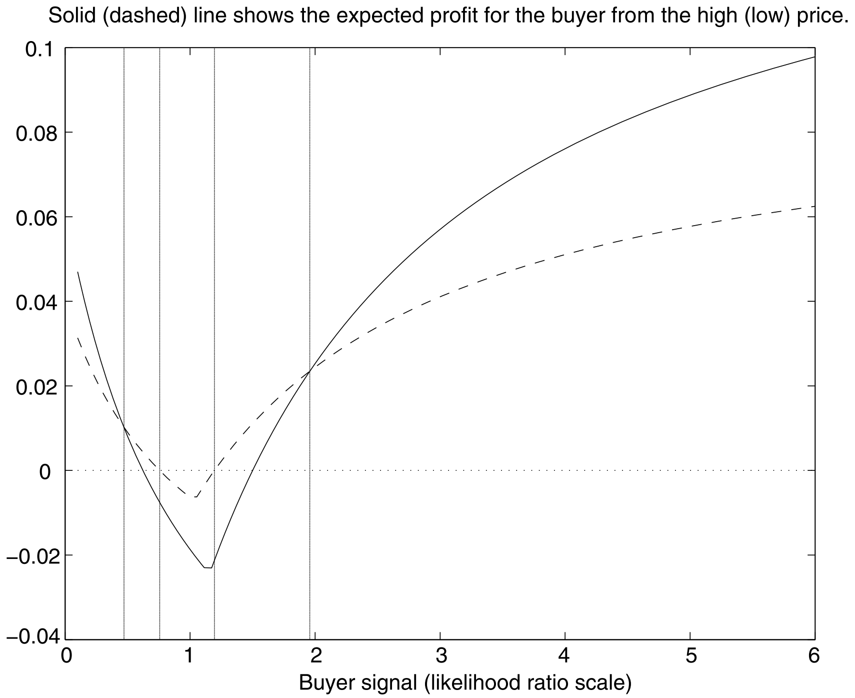

Fig. 2. The action set is $X = \{A, B\}$. The asset payoffs are $v(A, a) = 2$, $v(B, b) = 1$, and $v(A, b) = v(B, A) = 0$. The prices are $p^C = 14/18$ and $p^D = 13/18$ (note that $\min V = 12/18$). The *ex ante* probability of fundamental a is $1/2$. Both the buyer and seller observe a normally distributed signal with standard deviation 1 (for both fundamentals a, b) and a mean of 0 (respectively, 1) when the fundamental is b (respectively, a). The solid (dashed) line shows the buyer's expected profit from making the offer p^C (respectively, p^D). The vertical lines are drawn at $\underline{s}_2^C, \underline{s}_2^D, \bar{s}_2^D, \bar{s}_2^C$ respectively.

Let p denote the unique buyer's offer that is accepted, and let S_1 denote the set of signals at which the seller accepts this offer and sells. It follows that the buyer offers p if and only if

$$V^\ell\big(L_1(S_1)L_2(s_2)\big) \geqslant p.$$

Since the seller accepts the offer p with strictly positive probability, $p > \min V^\ell$. It follows that there exist $\underline{s}_2$ and $\bar{s}_2$ such that the buyer offers p if $s_2 \in (-\infty, \underline{s}_2) \cup (\bar{s}_2, \infty)$, with

$$V^\ell\big(L_1(S_1)L_2(\underline{s}_2)\big) = V^\ell\big(L_1(S_1)L_2(\bar{s}_2)\big) = p.$$

So the buyer makes zero profits when he sees $s_2 \in \{\underline{s}_2, \bar{s}_2\}$, regardless of whether or not he offers p at these signals. Moreover, observe that V^ℓ is decreasing at $L_1(S_1)L_2(\underline{s}_2)$ and increasing at $L_1(S_1)L_2(\bar{s}_2)$.

To prove (I), suppose to the contrary that $p > p^*$, and consider the deviation in which the buyer offers p^*. We show that this deviation is strictly profitable for the buyer at at least one of the signals $\underline{s}_2, \bar{s}_2$. The seller's beliefs are completely summarized by $\tilde{L}_2$. The seller then sells if $s_1 \in \tilde{S}_1$. If $L_1(\tilde{S}_1) > L_1(S_1^T)$ the buyer has a profitable deviation at $\bar{s}_2$. If $L_1(\tilde{S}_1) < L_1(S_1^T)$ the buyer has a profitable deviation at $\underline{s}_2$. If $L_1(\tilde{S}_1) = L_1(S_1^T)$ the buyer has a profitable deviation at both $\underline{s}_2$ and $\bar{s}_2$. In each case we have shown that p is not a best response for the buyer, giving a contradiction. To complete the proof, simply note that if $p < p^*$ then $p \leqslant \min V$ and the seller accepts the offer with zero probability.

To prove (II), let S_2^T denote the set of buyer signals at which the buyer offers p, and suppose to the contrary that the trade probability remains bounded away from 0 as $p^* \to \min V$. It follows that $L_2(S_2^T)$ is also bounded away from 0. The seller sells at signal s_1 only if $p \geqslant V^\ell(L_1(s_1)L_2(S_2^T))$. This occurs only if $L_1(s_1)L_2(S_2^T) \in (L^* - \varepsilon, L^* + \varepsilon)$ for some ε, where $\varepsilon \to 0$ as $p^* \to \min V$. Since $L_2(S_2^T)$ is bounded away from 0 it follows that $\Pr(s_1 \in S_1^T)$ converges to 0 as $p^* \to \min V$, which gives a contradiction and completes the proof.

Supplementary material

The online version of this article contains additional supplementary material.
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