Online appendix to *Information-based trade*

Philip Bond

Hülya Eraslan

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In the main text, we use a simple third-party posted price mechanism to establish sufficient conditions for trade. In this appendix, we analyze trade in a mechanism in which trading parties themselves set the price. Specifically, we consider the following trading mechanism: (1) the buyer proposes a price $p \in P$, where P is finite set of possible offers,[29] and (2) the seller accepts or rejects. We assume that P contains at least one element lying between $\min V$ and $\min \{V(0), V(1)\} - \delta$; and moreover that $0 \in P$, so that the buyer can effectively abstain from making an offer by offering the zero price.

Our first result shows that an equilibrium with meaningful trade is necessarily more complicated when the buyer chooses the price than when the price is simply imposed non-strategically. To see this, start by noting that in the equilibrium of Proposition 5 both the buyer and the seller make zero profit at the boundaries of their trade sets S_1^T and S_2^T. This is a consequence of continuity: for example, for the buyer, when $s_2 \in S_2^T$ the buyer's valuation of the asset exceeds p, while when $s_2 \notin S_2^T$ the price p exceeds the buyer's valuation.

Because the buyer has zero profits at the boundary signals $\underline{s}_2$ and $\bar{s}_2$ of S_2^T, he faces a strong temptation to offer a lower price after observing these signals. In fact, regardless of the seller's out-of-equilibrium beliefs the buyer could make strictly positive profits at at least one of $\underline{s}_2$ and $\bar{s}_2$ by offering $\tilde{p}$ between p and $\min V$. The reason is that the seller's response to $\tilde{p}$ either increases or decreases the buyer's belief that the fundamental is a relative to his equilibrium belief, and given the convex and

[29]The assumption that the offer set P is finite ensures that the action set is finite. As is well-known, equilibrium existence is not guaranteed in games with infinite action spaces.

non-monotone shape of V this raises the buyer's valuation at one of $\underline{s}_2$ and $\bar{s}_2$.

It follows that the only possibility for an equilibrium with trade at just one price entails trade at the lowest price in P that still exceeds $\min V$, i.e.,

$$p^* \equiv \min \{p \in P : p > \min V\}.$$

However, the seller only accepts this low price when he sees a signal that places his valuation between $\min V$ and p^*. When p^* is close to $\min V$ this can occur only rarely. Formally:

Proposition 8. *Suppose V is non-monotone, never flat,*[30] *and the buyer-posted price mechanism is used. Suppose an equilibrium exists in which trade occurs at only one price. Then (I) the trade price is p^*, and (II) the probability of trade approaches zero as $p^* \to \min V$.*

Proposition 8 implies that when the buyer chooses the price trade can occur with a meaningful probability only if the buyer makes different offers after different signals, and the seller accepts multiple offers with different probabilities. Characterizing such an equilibrium is challenging. Formally, our environment is close to a bargaining game with interdependent values and two-sided asymmetric information,[31] and we are not aware of any paper to consider such a game with more than two types.[32]

In order to illustrate the possibilities for trade when the buyer proposes the price, we focus on the simplest environment in which strategic offers by the buyer are possible, namely that in which V is non-monotone and the offer set is $P = \left\{p^C, p^D, 0\right\}$, where $\min \{V(0), V(1)\} - \delta > p^C > p^D > \min V$. As we explain below, even here the

[30]That is, there is no interval over which V is constant.

[31]However, our environment is *not* a special case of such a bargaining game because the value of the asset is endogenous.

[32]Schweizer (1989) analyzes the case of two types. In general, the literature on bargaining with interdependent values is small and focuses on the case of one-sided asymmetric information: see Evans (1989), Vincent (1989), Deneckere and Liang (2006), Dal Bo and Powell (2007).

fact that there are a continuum of buyer "types" necessitates a numerical simulation in order to verify the incentive constraints.

A trade equilibrium with trade at both prices p^C and p^D is characterized by signal sets S_2^C and S_2^D in which the buyer offers the prices p^C and p^D respectively, with corresponding signal sets S_1^C and S_1^D in which the seller accepts these offers. Given that V is convex and non-monotone the seller's acceptance sets S_1^C and S_1^D must be of the form $\left(\underline{s}_1^C, \bar{s}_1^C\right)$ and $\left(\underline{s}_1^D, \bar{s}_1^D\right)$. As in the equilibrium of Proposition 5, the seller has zero profits at the boundaries of S_1^C and S_1^D:

$$V\left(\Pr\left(a|\underline{s}_1^j, s_2 \in S_2^j\right)\right) = V\left(\Pr\left(a|\bar{s}_1^j, s_2 \in S_2^j\right)\right) = p^j \text{ for } j = C, D. \tag{22}$$

Since both p^C and p^D exceed $\min V$, there must exist a non-empty subset of signals at which the buyer prefers to make no offer. Again by the convexity and non-monotonicity of V, this no-offer set is an interval of the form $\left[\underline{s}_2^D, \bar{s}_2^D\right]$. Thus the equilibrium conditions for the buyer are that for $s_2 \in S_2^C$

$$\begin{aligned}&\Pr\left(s_1 \in S_1^C|s_2\right)\left(V\left(\Pr\left(a|s_1 \in S_1^C, s_2\right)\right) - \delta - p^C\right)\\ &\geq \max\left\{0, \Pr\left(s_1 \in S_1^D|s_2\right)\left(V\left(\Pr\left(a|s_1 \in S_1^D, s_2\right)\right) - \delta - p^D\right)\right\};\end{aligned} \tag{23}$$

while for $s_2 \in S_2^D$,

$$\begin{aligned}&\Pr\left(s_1 \in S_1^D|s_2\right)\left(V\left(\Pr\left(a|s_1 \in S_1^D, s_2\right)\right) - \delta - p^D\right)\\ &\geq \max\left\{0, \Pr\left(s_1 \in S_1^C|s_2\right)\left(V\left(\Pr\left(a|s_1 \in S_1^C, s_2\right)\right) - \delta - p^C\right)\right\};\end{aligned} \tag{24}$$

and if $s_2 \in \left[\underline{s}_2^D, \bar{s}_2^D\right]$,

$$0 \geq \max\left\{V\left(\Pr\left(a|s_1 \in S_1^C, s_2\right)\right) - \delta - p^C, V\left(\Pr\left(a|s_1 \in S_1^D, s_2\right)\right) - \delta - p^D\right\}.$$

Finally, in light of the form the equilibrium takes in the third-party mechanism, along with the non-monotonicity of V, a natural conjecture for the form of the buyer's trade sets S_2^C and S_2^D is as follows. The buyer makes the higher offer p^C only when his signal is relatively informative, that is, S_2^C is of the form $\mathbb{R} \backslash \left[\underline{s}_2^C, \bar{s}_2^C\right]$. The buyer then makes the lower offer p^D after the remaining signals $\mathbb{R} \backslash \left(S_2^C \cup \left[\underline{s}_2^D, \bar{s}_2^D\right]\right)$, so that $S_2^D = [\underline{s}_2^C, \underline{s}_2^D) \cup (\bar{s}_2^D, \bar{s}_2^C]$. By continuity, the following equalities are then *necessary* for an equilibrium of this type to exist:

$$V\left(\Pr\left(a|s_1 \in S_1^D, s_2\right)\right) - \delta = p^D \tag{25}$$

at $s_2 = \underline{s}_2^D, \bar{s}_2^D$; while at $s_2 = \underline{s}_2^C, \bar{s}_2^C$,

$$\begin{aligned} &\Pr\left(s_1 \in S_1^C|s_2\right)\left(V\left(\Pr\left(a|s_1 \in S_1^C, s_2\right)\right) - \delta - p^C\right) \\ &= \Pr\left(s_1 \in S_1^D|s_2\right)\left(V\left(\Pr\left(a|s_1 \in S_1^D, s_2\right)\right) - \delta - p^D\right). \end{aligned} \tag{26}$$

Together, equations (22), (25) and (26) constitute eight equations in the eight parameters $\left\{\underline{s}_1^C, \bar{s}_1^C, \underline{s}_1^D, \bar{s}_1^D, \underline{s}_2^C, \bar{s}_2^C, \underline{s}_2^D, \bar{s}_2^D\right\}$ that characterize the equilibrium of the type described.

Figure 2 displays an example of such an equilibrium for a specific set of parameter values.[33] The figure plots the buyer's expected profits from each of the offers p^C and p^D as a function of his signal s_2. The vertical lines are drawn at the boundaries of the sets S_2^C and S_2^D, i.e., at $\underline{s}_2^C, \underline{s}_2^D, \bar{s}_2^C, \bar{s}_2^D$ respectively. The figure makes clear that the solution to equations (22), (25) and (26) defines an equilibrium in this case: whenever $s_2 \in S_2^C$ the buyer prefers offering p^C to p^D, and whenever $s_2 \in S_2^D$ the buyer prefers offering p^D to p^C. The advantage of the higher offer p^C is that it is accepted more often; the disadvantage is, of course, that the buyer pays more.

The significance of this example relative to Proposition 8 is that it shows that trade can occur with significant probability when the buyer proposes different prices after different signals. That is, although the probability of trade at the lower price p^D is relatively low, the buyer sometimes offers the higher price p^C, and the seller's

[33]Unfortunately, we have not been able to characterize the conditions under which equations (22), (25) and (26) have a solution; and even if we were to do so, one would still need to check that the equilibrium conditions (23) and (24) hold away from the boundaries of the S_2^C and S_2^D.

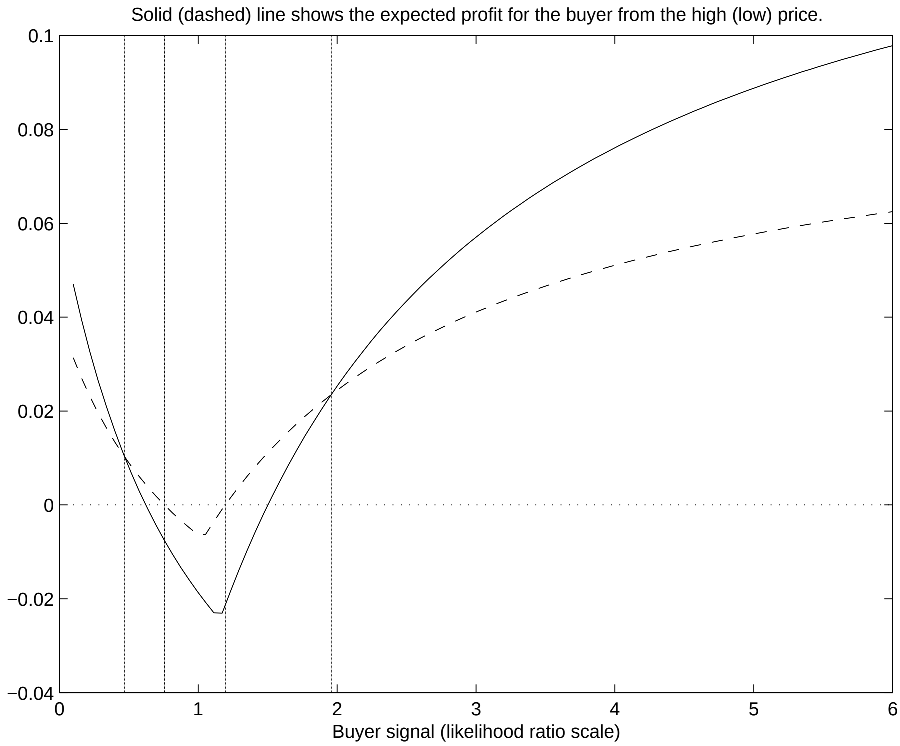

Figure 2: The asset value is as in the opening example. The prices are $p^C = 14/18$ and $p^D = 13/18$, and $\min V = 12/18$. Also as in the opening example the *ex ante* probability of fundamentals a is 1/2. Both the buyer and seller observe a normally distributed signal with standard deviation 1 (for both fundamentals a, b) and a mean of 0 (respectively, 1) when the fundamental is b (respectively, a). The solid (dashed) line shows the buyer's expected profit from making the offer p^C (respectively, p^D). The vertical lines are drawn at $\underline{s}_2^C, \underline{s}_2^D, \bar{s}_2^D, \bar{s}_2^C$ respectively.

acceptance probability at this price is higher. Moreover, our focus on an equilibrium with trade at two prices is solely for tractability, and we conjecture that with a finer offer set P there exist equilibria with trade at a large number of different prices. We leave the further exploration of this bargaining framework for future research.

References

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Proof of Proposition 8

Let p denote the unique buyer's offer that is accepted, and let S_1 denote the set of signals at which the seller accepts this offer and sells. It follows that the buyer offers p if and only if

$$V^{\ell}\left(L_1\left(S_1\right)L_2\left(s_2\right)\right)-\delta\geq p,$$

where the likelihood ratios L_i and function V^ℓ are as defined in the proof of Proposition 5. From Proposition 1 and the proof of Proposition 5, we know that there exists L^* such that V^ℓ is strictly decreasing over $[0, L^*)$ and strictly increasing over (L^*, ∞). Since the seller accepts the offer p with strictly positive probability, $p > \min V^\ell$. It follows that there exist $\underline{s}_2$ and $\bar{s}_2$ such that the buyer offers p if $s_2 \in (-\infty, \underline{s}_2) \cup (\bar{s}_2, \infty)$, with

$$V^\ell \left(L_1 \left(S_1\right) L_2 \left(\underline{s}_2\right)\right) = V^\ell \left(L_1 \left(S_1\right) L_2 \left(\bar{s}_2\right)\right) = p + \delta.$$

So the buyer makes zero profits when he sees $s_2 \in \{\underline{s}_2, \bar{s}_2\}$, regardless of whether or not he offers p at these signals. Moreover, observe that V^ℓ is decreasing at $L_1 \left(S_1\right) L_2 \left(\underline{s}_2\right)$ and increasing at $L_1 \left(S_1\right) L_2 \left(\bar{s}_2\right)$.

To prove (I), suppose to the contrary that $p > p^*$, and consider the deviation in which the buyer offers p^*. We show that this deviation is strictly profitable for the buyer at at least one of the signals $\underline{s}_2, \bar{s}_2$. The seller's beliefs are completely summarized by $\tilde{L}_2$. The seller then sells if $s_1 \in \tilde{S}_1$. If $L_1\left(\tilde{S}_1\right) > L_1\left(S_1^T\right)$ the buyer has a profitable deviation at $\bar{s}_2$. If $L_1\left(\tilde{S}_1\right) < L_1\left(S_1^T\right)$ the buyer has a profitable deviation at $\underline{s}_2$. If $L_1\left(\tilde{S}_1\right) = L_1\left(S_1^T\right)$ the buyer has a profitable deviation at both $\underline{s}_2$ and $\bar{s}_2$. In each case we have shown that p is not a best response for the buyer, giving a contradiction. To complete the proof, simply note that if $p < p^*$ then $p \leq \min V$ and the seller accepts the offer with zero probability.

To prove (II), let S_2^T denote the set of buyer signals at which the buyer offers p, and suppose to the contrary that the trade probability remains bounded away from 0 as $p^* \to \min V$. It follows that $L_2\left(S_2^T\right)$ is also bounded away from 0. The seller sells at signal s_1 only if $p \geq V^\ell\left(L_1\left(s_1\right) L_2\left(S_2^T\right)\right)$. This occurs only if $L_1\left(s_1\right) L_2\left(S_2^T\right) \in (L^* - \varepsilon, L^* + \varepsilon)$ for some ε, where $\varepsilon \to 0$ as $p^* \to \min V$. Since $L_2\left(S_2^T\right)$ is bounded away from 0 it follows that $\Pr\left(s_1 \in S_1^T\right)$ converges to 0 as $p^* \to \min V$, which gives a contradiction and completes the proof. ∎