



# Efficiency of flexible budgetary institutions <sup>☆</sup>

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## Abstract

Which budgetary institutions result in efficient provision of public goods? We analyze a model with two parties bargaining over the allocation to a public good each period. Parties place different values on the public good, and these values may change over time. We focus on budgetary institutions that determine the rules governing feasible allocations to mandatory and discretionary spending programs. Mandatory spending is enacted by law and remains in effect until changed, and thus induces an endogenous status quo, whereas discretionary spending is a periodic appropriation that is not allocated if no new agreement is reached. We show that discretionary only and mandatory only institutions typically lead to dynamic inefficiency and that mandatory only institutions can even lead to static inefficiency. By introducing appropriate flexibility in mandatory programs, we obtain static and dynamic efficiency. This flexibility is provided by an endogenous choice of mandatory and discretionary programs, sunset provisions and state-contingent mandatory programs in increasingly complex environments.

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## 1. Introduction

Allocation of resources to public goods is typically decided through budget negotiations. In many democratic governments these negotiations occur annually and are constrained by the *budgetary institutions* in place. In designing budgetary institutions one may have various goals, such as efficiency, responsiveness to citizens' preferences, or accountability. There has been increasing interest among policy-makers in understanding how to achieve these goals in both developed and developing countries (see, for example, Santiso, 2006; Shah, 2007).<sup>1</sup> Economic research has also recognized the importance of budgetary institutions (see, for example, Hallerberg et al., 2009).<sup>2</sup> These studies emphasize the importance of various dimensions of budgetary institutions including transparency and centralization of decision-making. We focus on a different dimension in this paper: the rules governing feasible allocations to mandatory and discretionary spending programs.<sup>3</sup> Discretionary programs require periodic appropriations, and no spending is allocated if no new agreement is reached. By contrast, mandatory programs are enacted by law, and spending continues into the future until changed. Thus under mandatory programs, spending decisions today determine the status quo level of spending for tomorrow.

Naturally, there may be disagreement on the appropriate level of public spending, and the final spending outcome is the result of negotiations between parties that represent different interests. Negotiations are typically led by the party in power whose identity may change over time, bringing about turnover in agenda-setting power. Bowen et al. (2014) show that in a stable economic environment, mandatory programs improve the efficiency of public good provision over discretionary programs by mitigating the inefficiency due to turnover. However, the economic environment may be changing over time, potentially resulting in evolving preferences. Hence, the party in power today must consider how current spending on the public good affects future spending when preferences and the agenda-setter are possibly different from today.

In this paper we address the question of which budgetary institutions result in efficient provision of public goods in an environment with disagreement over the value of the public good, changing economic conditions, and turnover in political power. In this environment it is natural to expect that rigid budgetary institutions that allow only discretionary or only mandatory spending fail to deliver efficiency. Indeed, in settings different from ours, Riboni and Ruge-Murcia

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<sup>1</sup> The OECD has devoted resources to surveying budget practices and procedures across countries since 2003. See *International Budget Practices and Procedures Database*, OECD (2012).

<sup>2</sup> See also Alesina and Perotti (1995) for a survey of the early literature recognizing the importance of budgetary institutions.

<sup>3</sup> This terminology is used in the United States budget. Related institutions exist in other budget negotiations, for example the budget of the European Union is categorized into commitment and payment appropriations. The main distinction is that one has dynamic consequences because agreements are made for future budgets, and the other does not.

(2008), Zápal (2011) and Dziuda and Loeper (2016) note that inefficiency can arise from mandatory only institutions when preferences are evolving.<sup>4</sup> In our setting we show that rigid budgetary institutions in general lead to inefficiencies. More importantly, we show that efficiency can be obtained when appropriate flexibility is incorporated into budgetary institutions. We show this in increasingly complex environments.

We begin by analyzing a model in which two parties with concave utility functions bargain over the spending on a public good in each of two periods. The parties place different values on the public good, and these values may change over time, reflecting changes in the underlying economic environment. To capture turnover in political power, we assume the proposing party is selected at random each period. Unanimity is required to implement the proposed spending on the public good. We investigate the efficiency properties of the equilibrium outcome of this bargaining game under different budgetary institutions.

We distinguish between *static Pareto efficiency* and *dynamic Pareto efficiency*. A statically Pareto efficient allocation in a given period is a spending level such that no alternative would make both parties better off and at least one of them strictly better off in that period. A dynamically Pareto efficient allocation is a sequence of spending levels, one for each period, that needs to satisfy a similar requirement except that the utility possibility frontier is constructed using the discounted sum of utilities. Dynamic efficiency puts intertemporal restrictions on spending levels in addition to requiring static efficiency for each period, making it a stronger requirement than static Pareto efficiency. Furthermore, due to the concavity of utility functions, any dynamically Pareto efficient allocation cannot involve randomization. This means that any equilibrium in which spending varies with the identity of the proposer (what we call *political risk*) cannot be dynamically Pareto efficient. We further show that when preferences evolve over time, dynamic Pareto efficiency typically requires that spending levels change accordingly, that is, dynamic efficiency requires that parties avoid *gridlock*.

Comparing equilibrium allocations with the efficient ones, we show that discretionary only institutions lead to static efficiency but dynamic inefficiency due to political risk. Specifically, since the status quo of a discretionary spending program is exogenously zero, the equilibrium level of spending varies with the party in power.

With mandatory only institutions, any equilibrium is dynamically inefficient because the second period's spending level either varies with the identity of the proposer, which leads to political risk, or is equal to the first period's level, which results in gridlock. Even static inefficiency may result with mandatory only institutions. This is because the parties' concerns about their future bargaining positions, which are determined by the first period's spending level, can lead the parties to reach an outcome that goes against their first-period interests.

In contrast, budgetary institutions that allow flexibility with a combination of discretionary and mandatory programs avoid both political risk and gridlock, resulting in dynamic efficiency.<sup>5</sup> This is true because the party in power in the first period finds it optimal to set the size of the mandatory program to a level that is statically efficient in the second period. Given this, the status quo is maintained in the second period regardless of which party comes into power, thereby eliminating political risk. The party in power in the first period can then use discretionary

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<sup>4</sup> We further discuss how our results relate to these and other papers at the end of the Introduction.

<sup>5</sup> Examples of budget functions in the United States with significant fractions of both mandatory and discretionary spending include income security, commerce and housing credit, and transportation (see [Budget of the United States Government, 2015](#)). Policymakers explicitly specify the budget enforcement act category, that is, mandatory or discretionary, when proposing changes to spending on budget functions (see, for example, [House Budget Committee, 2014](#)).

spending to tailor the total spending to the desired level in the first period, avoiding gridlock. The main insight is that the flexibility afforded by a combination of mandatory and discretionary programs delivers efficiency. However, this efficiency result breaks down with a longer time horizon because to eliminate political risk in all future periods, the first-period proposer must be able to set all future status quos independently, which is not feasible with a simple combination of mandatory and discretionary programs. In this case, we show that efficiency is achieved with sunset provisions with appropriately chosen expiration dates.

To extend our result to an even richer environment, we consider a model with an arbitrary time horizon and stochastic preferences that depend on the economic state. We analyze a budgetary institution in which proposers choose a spending rule that gives spending levels conditional on the realization of the state. We show that the first-period proposer chooses a rule that is dynamically efficient and once chosen, this spending rule is retained because no future proposer can make a different proposal that is better for itself and acceptable to the other party. Thus state-contingent mandatory programs allow sufficient flexibility to achieve dynamic efficiency, even though we consider spending rules that cannot condition on the proposer identity.

The use of state-contingent programs dates back to at least Ancient Egypt, where the rate of taxation depended on the extent of Nile flooding in any given year (see [Breasted, 1945](#), page 191). Such state-contingency can also be found in practice in modern economies as automatic adjustments embedded in mandatory programs. For example, in the United States unemployment insurance may fluctuate with the unemployment rate through “extended” or “emergency” benefits. These benefits have been a feature of the unemployment insurance law since 1971, and are triggered by recession on the basis of certain unemployment indicators (see [Nicholson and Needels, 2006](#)).<sup>6</sup> Similarly, in Canada the maximum number of weeks one can receive unemployment benefits depends on the local rate of unemployment (see [Canadian Minister of Justice, 2014](#), Schedule I, page 180). The efficiency of state-contingent spending programs may explain why they are successfully implemented in practice.

Our work is related to several strands of literature. A large body of political economy research studies efficiency implications of policies that arise in a political equilibrium.<sup>7</sup> As highlighted in [Besley and Coate \(1998\)](#) inefficiency can arise because policies either yield benefits in the future when the current political representation might not enjoy them, or alter the choices of future policy makers, or may change the probability of the current political representation staying in power. Our paper shares with the rest of the literature the first two sources of inefficiency, but unlike the rest of the literature, our main focus is on linking these sources of inefficiency to budgetary institutions that specify the rules governing feasible allocations to mandatory and discretionary spending programs.

Modeling mandatory spending programs as an endogenous status quo links our work to a growing dynamic bargaining literature.<sup>8</sup> With the exception of [Bowen et al. \(2014\)](#) and [Zápal \(2011\)](#) this literature has focused on studying models only with policies that have the endoge-

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<sup>6</sup> See also *Federal-State Extended Unemployment Compensation Act of 1970*, U.S. House of Representatives, Office of the Legislative Counsel (2013).

<sup>7</sup> See, for example, [Persson and Svensson \(1984\)](#), [Alesina and Tabellini \(1990\)](#), [Krusell and Ríos-Rull \(1996\)](#), [Dixit et al. \(2000\)](#), [Lizzeri and Persico \(2001\)](#), [Battaglini and Coate \(2007\)](#), [Acemoglu et al. \(2008, 2011\)](#), [Aguiar and Amador \(2011\)](#), [Azzimonti \(2011\)](#), [Bai and Lagunoff \(2011\)](#), [Van Weelden \(2013\)](#), [Callander and Krehbiel \(2014\)](#), [Bierbrauer and Boyer \(2016\)](#).

<sup>8</sup> This literature includes [Baron \(1996\)](#), [Kalandrakis \(2004, 2010\)](#), [Riboni and Ruge-Murcia \(2008\)](#), [Diermeier and Fong \(2011\)](#), [Zápal \(2011\)](#), [Battaglini and Palfrey \(2012\)](#), [Duggan and Kalandrakis \(2012\)](#), [Piguillem and Riboni \(2012, 2015\)](#), [Diermeier et al. \(2013\)](#), [Levy and Razin \(2013\)](#), [Baron and Bowen \(2014\)](#), [Bowen et al. \(2014\)](#), [Forand \(2014\)](#),

nous status quo property. In the language of our model, this literature has focused on mandatory spending programs only. Bowen et al. (2014) model discretionary and mandatory spending programs, but do not allow for an endogenous choice of these two types of programs. Moreover, unlike in their model, we allow the values parties attach to the public good to vary over time, which plays an important role in our results. Bowen et al. (2014) show that mandatory programs *ex-ante* Pareto dominate discretionary programs under certain conditions, whereas we show that with evolving preferences mandatory programs with appropriate flexibility achieve dynamic efficiency. Zápál (2011) demonstrates that a budgetary institution that allows for distinct current-period policy and future-period status quo eliminates static inefficiency. This result parallels the efficiency of an endogenous choice of mandatory and discretionary programs that we show, but we do this in an environment with political turnover and arbitrary variation in preferences. Furthermore, we also demonstrate the efficiency of state-contingent mandatory programs in this richer setting.

Our focus on budgetary institutions connects our work to papers studying fiscal rules and fiscal constitutions.<sup>9</sup> This literature has focused on other fiscal rules or constitutions, for example, constraints on government spending and taxation, limits on public debt or deficits, or decentralization of spending authority.

In the next section we present our model and in Section 3 we discuss Pareto efficient allocations and equilibria. In Section 4 we study rigid budgetary institutions, specifically mandatory only and discretionary only, and discuss why these lead to inefficiencies. In Section 5 we study flexible budgetary institutions, specifically an endogenous choice of mandatory and discretionary, sunset provisions, state-contingent mandatory, and show that these lead to efficiency in increasingly complex environments. We conclude in Section 6. All proofs omitted in the main text are in the Appendix.

## 2. Model

Consider a stylized economy and political system with two parties labeled *A* and *B*. There are two time periods indexed by  $t \in \{1, 2\}$ .<sup>10</sup> In each period  $t$ , the two parties decide an allocation to a public good  $x_t \in \mathbb{R}_+$ . The stage utility for party  $i \in \{A, B\}$  in period  $t$  is  $u_{it}(x_t)$ . Party  $i$  seeks to maximize its dynamic payoff from the sequence of public good allocations  $u_{i1}(x_1) + \delta u_{i2}(x_2)$ , where  $\delta \in (0, 1]$  is the parties' common discount factor. We assume  $u_{it}$  is twice continuously differentiable, strictly concave, and attains a maximum at  $\theta_{it} > 0$  for all  $i \in \{A, B\}$  and  $t \in \{1, 2\}$ . This implies  $u_{it}$  is single-peaked with  $\theta_{it}$  denoting party  $i$ 's ideal level of the public good in period  $t$ .<sup>11</sup>

We consider a political system with unanimity rule.<sup>12</sup> At the beginning of each period, a party is randomly selected to make a proposal for the allocation to the public good. The probability

Ma (2014), Anesi and Seidmann (2015), Nunnari and Zápál (2015), Chen and Eraslan (2016), Dziuda and Loeper (2016), Kalandrakis (2016).

<sup>9</sup> See, for example, Persson and Tabellini (1996a, 1996b), Stockman (2001), Besley and Coate (2003), Besley and Smart (2007), Caballero and Yared (2010), Yared (2010), Halac and Yared (2014), Azzimonti et al. (2016).

<sup>10</sup> In Section 5.3 we consider a more general model with any number of periods and random preferences.

<sup>11</sup> Because of the opportunity cost of providing public goods, it is reasonable to model parties' utility functions as single-peaked as in, for example, Baron (1996).

<sup>12</sup> Most political systems are not formally characterized by unanimity rule, however, many have institutions that limit a single party's power, for example, the "checks and balances" included in the U.S. Constitution. Under these institutions, if the majority party's power is not sufficiently high, then it needs approval of the other party to set new policies.

that party  $i$  proposes in a period is  $p_i \in (0, 1)$ .<sup>13</sup> Spending on the public good may be allocated by way of different programs—a discretionary program, which expires after one period, or a mandatory program, for which spending will continue in the next period unless the parties agree to change it. Denote the proposed amount allocated to a discretionary program in period  $t$  as  $k_t$ , and to a mandatory program as  $g_t$ . If the responding party agrees to the proposal, the implemented allocation to the public good for the period is the sum of the discretionary and mandatory allocations proposed, so  $x_t = k_t + g_t$ ; otherwise,  $x_t = g_{t-1}$ .

Denote a proposal by  $z_t = (k_t, g_t)$ . We require  $g_t \geq 0$  to ensure a positive status quo each period. Let  $Z \subseteq \mathbb{R} \times \mathbb{R}_+$  be the set of feasible proposals. The set  $Z$  is determined by the rules governing mandatory and discretionary programs, and hence we call  $Z$  the *budgetary institution*. We consider the following institutions: only discretionary programs, in which case  $Z = \mathbb{R}_+ \times \{0\}$ ; only mandatory programs, in which case  $Z = \{0\} \times \mathbb{R}_+$ ; and both mandatory and discretionary, where discretionary spending may be positive or negative, in which case  $Z = \{(k_t, g_t) \in \mathbb{R} \times \mathbb{R}_+ | k_t + g_t \geq 0\}$ . It is natural to think of spending as positive, but it is also possible to have temporary cuts in mandatory programs, for example government furloughs that temporarily reduce public employees' salaries. This temporary reduction in mandatory spending can be thought of as negative discretionary spending as it reduces spending in the current period without affecting the status quo for the next period.

A pure strategy for party  $i$  in period  $t$  is a pair of functions  $\sigma_{it} = (\pi_{it}, \alpha_{it})$ , where  $\pi_{it} : \mathbb{R}_+ \rightarrow Z$  is a proposal strategy for party  $i$  in period  $t$  and  $\alpha_{it} : \mathbb{R}_+ \times Z \rightarrow \{0, 1\}$  is an acceptance strategy for party  $i$  in period  $t$ .<sup>14</sup> Party  $i$ 's proposal strategy  $\pi_{it} = (\kappa_{it}, \gamma_{it})$  associates with each status quo  $g_{t-1}$  an amount of public good spending in discretionary programs, denoted by  $\kappa_{it}(g_{t-1})$ , and an amount in mandatory programs, denoted by  $\gamma_{it}(g_{t-1})$ . Party  $i$ 's acceptance strategy  $\alpha_{it}(g_{t-1}, z_t)$  takes the value 1 if party  $i$  accepts the proposal  $z_t$  offered by the other party when the status quo is  $g_{t-1}$ , and 0 otherwise.<sup>15</sup>

We consider subgame perfect equilibria and restrict attention to equilibria in which (i)  $\alpha_{it}(g_{t-1}, z_t) = 1$  when party  $i$  is indifferent between  $g_{t-1}$  and  $z_t$ ; and (ii)  $\alpha_{it}(g_{t-1}, \pi_{jt}(g_{t-1})) = 1$  for all  $t$ ,  $g_{t-1} \in \mathbb{R}_+$ ,  $i, j \in \{A, B\}$  with  $j \neq i$ . That is, the responder accepts any proposal that it is indifferent between accepting and rejecting, and the equilibrium proposals are always accepted.<sup>16</sup> We henceforth refer to a subgame perfect equilibrium that satisfies (i) and (ii) simply as an equilibrium.

Denote an equilibrium by  $\sigma^*$ . Let party  $i \in \{A, B\}$  be the proposer and party  $j \in \{A, B\}$  be the responder in period 2. (When we use  $i$  to denote the proposer and  $j$  to denote the respon-

<sup>13</sup> More generally, the probability that party  $i$  proposes in period  $t$  is  $p_{it}$ . In the two-period model,  $p_{i1}$  does not play a role, and for notational simplicity we write  $p_i$  as the probability that party  $i$  proposes in period 2. In Section 5.3 we extend our model to an arbitrary time horizon and we use the general notation.

<sup>14</sup> In this two-period model, we show that equilibrium spending (conditional on proposer) in the second period is unique. Thus, in equilibrium, the second-period strategy does not depend on the history except through the status quo, so writing strategies as depending on history only through the status quo is without loss of generality. This result extends to the finite-horizon case of state-contingent mandatory spending considered in Section 5.3. For the infinite-horizon case, the restriction on strategies implies a Markov restriction on the equilibrium.

<sup>15</sup> We are interested in efficiency properties of budgetary institutions. Because the utility functions are strictly concave, Pareto efficient allocations do not involve randomization. Hence, if any pure strategy equilibrium is inefficient, allowing mixed strategies does not improve efficiency.

<sup>16</sup> Any equilibrium is payoff equivalent to some equilibrium (possibly itself) that satisfies (i) and (ii). Similar restrictions are made in Bowen et al. (2014) and the proof follows the same arguments as in that paper. We omit the arguments here for space considerations.

der without any qualifier, it is understood that  $i \neq j$ .) Given conditions (i) and (ii), for any  $g_1$  admissible under  $Z$ , the equilibrium proposal strategy  $(\kappa_{i2}^*(g_1), \gamma_{i2}^*(g_1))$  of party  $i$  in period 2 solves

$$\begin{aligned} \max_{(k_2, g_2) \in Z} & u_{i2}(k_2 + g_2) \\ \text{s.t.} & u_{j2}(k_2 + g_2) \geq u_{j2}(g_1). \end{aligned} \tag{P_2}$$

Let  $V_i(g; \sigma_2)$  be the expected second-period payoff for party  $i$  given first-period mandatory spending  $g$  and second-period strategies  $\sigma_2 = (\sigma_{A2}, \sigma_{B2})$ . That is

$$V_i(g; \sigma_2) = p_A u_{i2}(\kappa_{A2}(g) + \gamma_{A2}(g)) + p_B u_{i2}(\kappa_{B2}(g) + \gamma_{B2}(g)).$$

If party  $i$  is the proposer and party  $j$  is the responder in period 1, then for any  $g_0$  admissible under  $Z$  the equilibrium proposal strategy  $(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0))$  of party  $i$  in period 1 solves

$$\begin{aligned} \max_{(k_1, g_1) \in Z} & u_{i1}(k_1 + g_1) + \delta V_i(g_1; \sigma_2^*) \\ \text{s.t.} & u_{j1}(k_1 + g_1) + \delta V_j(g_1; \sigma_2^*) \geq u_{j1}(g_0) + \delta V_j(g_0; \sigma_2^*). \end{aligned} \tag{P_1}$$

### 3. Pareto efficiency

In this section we characterize Pareto efficient allocations and define Pareto efficient equilibria, both in the static and dynamic sense.

#### 3.1. Pareto efficient allocations

We distinguish between the social planner’s static problem (SSP), which determines static Pareto efficient allocations, and the social planner’s dynamic problem (DSP), which determines dynamic Pareto efficient allocations.

We define a statically Pareto efficient allocation in period  $t$  as the solution to the following maximization problem

$$\begin{aligned} \max_{x_t \in \mathbb{R}_+} & u_{it}(x_t) \\ \text{s.t.} & u_{jt}(x_t) \geq \bar{u} \end{aligned} \tag{SSP}$$

for some  $\bar{u} \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ .<sup>17</sup> By Proposition 1, statically Pareto efficient allocations are all those between the ideal points of the parties. To write the proposition, let  $\underline{\theta}_t = \min\{\theta_{At}, \theta_{Bt}\}$  and  $\bar{\theta}_t = \max\{\theta_{At}, \theta_{Bt}\}$ .

**Proposition 1.** *An allocation  $x_t$  is statically Pareto efficient in period  $t$  if and only if  $x_t \in [\underline{\theta}_t, \bar{\theta}_t]$ .*

Denote a sequence of allocations by  $\mathbf{x} = (x_1, x_2)$  and party  $i$ ’s discounted dynamic payoff from  $\mathbf{x}$  by  $U_i(\mathbf{x}) = \sum_{t=1}^2 \delta^{t-1} u_{it}(x_t)$ . We define a dynamically Pareto efficient allocation as the solution to the following maximization problem

<sup>17</sup> The social planner’s static problem (SSP) is a standard concave programming problem so the solution is unique for a given  $\bar{u}$  if it exists.



$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}_+^2} U_i(\mathbf{x}) \\ \text{s.t. } U_j(\mathbf{x}) \geq \bar{U} \end{aligned} \tag{DSP}$$

for some  $\bar{U} \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ . In (DSP) we only allow deterministic allocations. Allowing for randomization, possibly depending on proposer identity, would not change the solution to (DSP) due to the strict concavity of the utility functions.

Denote the sequence of party  $i$ 's static ideals by  $\theta_i = (\theta_{i1}, \theta_{i2})$  for all  $i \in \{A, B\}$ , and denote the solution to (DSP) as  $\mathbf{x}^* = (x_1^*, x_2^*)$ .<sup>18</sup> Proposition 2 characterizes the dynamically Pareto efficient allocations.<sup>19</sup>

**Proposition 2.** *A dynamically Pareto efficient allocation  $\mathbf{x}^*$  satisfies the following properties:*

1. For all  $t$ ,  $x_t^*$  is statically Pareto efficient. That is,  $x_t^* \in [\underline{\theta}_t, \bar{\theta}_t]$  for all  $t$ .
2. Either  $\mathbf{x}^* = \theta_A$ , or  $\mathbf{x}^* = \theta_B$ , or  $u'_{At}(x_t^*) + \lambda^* u'_{Bt}(x_t^*) = 0$  for some  $\lambda^* > 0$ , for all  $t$ .

Proposition 2 part 2 implies that if  $\mathbf{x}^* \neq \theta_i$  for all  $i \in \{A, B\}$ , and  $\theta_{At} \neq \theta_{Bt}$  in period  $t$  then we must have

$$-\frac{u'_{At}(x_t^*)}{u'_{Bt}(x_t^*)} = \lambda^* \tag{1}$$

for some  $\lambda^* > 0$ .<sup>20</sup> By (1) if parties  $A$  and  $B$  do not have the same ideal level of the public good in periods 1 and 2, then in a dynamically Pareto efficient allocation, either the allocation is equal to party  $A$ 's or party  $B$ 's ideal in both periods, or the ratio of their marginal utilities is equal across these two periods, i.e.,  $\frac{u'_{A1}(x_1^*)}{u'_{B1}(x_1^*)} = \frac{u'_{A2}(x_2^*)}{u'_{B2}(x_2^*)}$ . The intuition for the latter is that if the ratio of marginal utilities is not constant across periods, then there is an intertemporal reallocation such that at least one party is strictly better off and the other party is no worse off. In both cases there is a *dynamic link* across periods.

### 3.2. Pareto efficient equilibrium

We define a dynamically Pareto efficient equilibrium given an initial status quo  $g_0$  as an equilibrium such that, conditional on the realization of the first-period proposer, the resulting allocation is a solution to (DSP). As discussed before, the solution to (DSP) does not involve any randomization. Hence, the resulting allocation needs to be both dynamically efficient and independent of the identity of the second-period proposer.

More precisely, denote an equilibrium strategy profile as  $\sigma^* = ((\sigma_{A1}^*, \sigma_{A2}^*), (\sigma_{B1}^*, \sigma_{B2}^*))$  with  $\sigma_{it}^* = ((\kappa_{it}^*, \gamma_{it}^*), \alpha_{it}^*)$ . An *equilibrium allocation* for  $\sigma^*$  given initial status quo  $g_0$  is a possible realization of total public good spending for each period  $\mathbf{x}^{\sigma^*}(g_0) = (x_1^{\sigma^*}(g_0), x_2^{\sigma^*}(g_0))$ , where  $x_1^{\sigma^*}(g_0) = \kappa_{i1}^*(g_0) + \gamma_{i1}^*(g_0)$ , and  $x_2^{\sigma^*}(g_0) = \kappa_{j2}^*(\gamma_{i1}^*(g_0)) + \gamma_{j2}^*(\gamma_{i1}^*(g_0))$  for some  $i, j \in \{A, B\}$ .

<sup>18</sup> Note the solution to (DSP) depends on  $\bar{U}$ , but for notational simplicity we suppress this dependency and denote the solution to (DSP) as  $\mathbf{x}^*$ . The solution to (DSP) is unique for a given  $\bar{U}$  if it exists.

<sup>19</sup> In the proof of Proposition 2 in the Appendix, we generalize (DSP) to any number of periods and prove Proposition 2 for this more general problem.

<sup>20</sup> This is because if  $u'_{Bt}(x_t^*) = 0$ , then part 2 of Proposition 2 implies that we must also have  $u'_{At}(x_t^*) = 0$  which is not possible when  $\theta_{At} \neq \theta_{Bt}$ .



The random determination of proposers induces a probability distribution over allocations given an equilibrium  $\sigma^*$ . Thus any element in the support of this distribution is an equilibrium allocation for  $\sigma^*$ .<sup>21</sup>

**Definition 1.** An equilibrium  $\sigma^*$  is a dynamically Pareto efficient equilibrium given initial status quo  $g_0$  if and only if

1. every equilibrium allocation  $\mathbf{x}^{\sigma^*}(g_0)$  is dynamically Pareto efficient; and
2. given the first-period proposer, every equilibrium allocation  $\mathbf{x}^{\sigma^*}(g_0)$  is identical.

A statically Pareto efficient equilibrium given initial status quo  $g_0$  is analogously defined as an equilibrium in which the realized allocation to the public good is statically Pareto efficient in all periods  $t$  given initial status quo  $g_0$ . Thus a necessary condition for  $\sigma^*$  to be a dynamically Pareto efficient equilibrium is that  $\sigma^*$  is a statically Pareto efficient equilibrium.

Definition 1 part 2 implies that if the equilibrium level of spending in period 2 varies with the identity of the period-2 proposer, then the equilibrium cannot be dynamically Pareto efficient. Thus a dynamically Pareto efficient equilibrium avoids *political risk*.<sup>22</sup>

#### 4. Rigid budgetary institutions

In this section we show that budgetary institutions that allow only discretionary spending or only mandatory spending lead to Pareto inefficiency in general. For expositional simplicity, in this section, we assume  $u_{it}(x_t) = -(x_t - \theta_{it})^2$ ,  $\theta_{i1} \neq \theta_{i2}$  and  $\theta_{A1} < \theta_{B1}$  for all  $i$  and  $t$ . As explained above, dynamic Pareto efficiency avoids political risk. We next show that it also generically requires variation in spending across periods, which we call avoiding *gridlock*.

**Lemma 1.** There is a continuum of dynamically Pareto efficient allocations and at most one with a constant spending level across periods, that is, there is at most one  $\mathbf{x}^*$  with  $x_1^* = x_2^*$ .

For quadratic utilities, an allocation  $\mathbf{x}$  is dynamically Pareto efficient if and only if  $\mathbf{x} = \alpha\boldsymbol{\theta}_A + (1 - \alpha)\boldsymbol{\theta}_B$  for some  $\alpha \in [0, 1]$  and thus the lemma follows when the parties' ideals differ and evolve over time. To establish dynamic Pareto inefficiency of equilibria under the budgetary institutions considered in this section, we show that they display either political risk or gridlock.

<sup>21</sup> For example, if  $A$  is the proposer in period 1 and  $B$  is the proposer in period 2, then the equilibrium allocation is  $x_1^{\sigma^*}(g_0) = \kappa_{A1}^*(g_0) + \gamma_{A1}^*(g_0)$ , and  $x_2^{\sigma^*}(g_0) = \kappa_{B2}^*(\gamma_{A1}^*(g_0)) + \gamma_{B2}^*(\gamma_{A1}^*(g_0))$ .

<sup>22</sup> Note that our definition of a dynamically Pareto efficient equilibrium requires *interim* dynamic Pareto efficiency, that is, allocations must be dynamically Pareto efficient after the realization of the first-period proposer but before the realization of the second-period proposer. An alternative notion is *ex-ante* dynamic Pareto efficiency, before the realization of the first-period proposer. This notion would require the first-period allocation to be invariant to the party in power, a stronger requirement than *interim* Pareto efficiency. The working paper version of our paper (Bowen et al., 2016) considers *ex-post* dynamic Pareto efficiency, which requires that for each realized path of proposers the equilibrium allocation is dynamically Pareto efficient. As we show in the working paper, *interim* and *ex-post* Pareto efficiency coincide as long as  $\theta_{At} \neq \theta_{Bt}$  for all  $t$  and hence our results remain the same.

#### 4.1. Discretionary spending only

Suppose spending is allocated through discretionary programs only, that is,  $Z = \mathbb{R}_+ \times \{0\}$ . Since  $g_t$  is zero for any  $t$ , for the rest of this subsection we denote a proposal in period  $t$  by  $k_t$ . Since there is no dynamic link between periods, the bargaining between the two parties each period is a static problem, similar to the monopoly agenda-setting model in [Romer and Rosenthal \(1978, 1979\)](#). Consider any period  $t$ . Since  $u_{Bt}(0) < u_{Bt}(\theta_{At})$ , it follows that if party  $A$  is the proposer, it proposes its ideal policy  $k_t = \theta_{At}$ , and party  $B$  accepts. If party  $B$  is the proposer, it proposes  $k_t = \min\{\theta_{Bt}, 2\theta_{At}\} \in (\theta_{At}, \theta_{Bt}]$ . To see this, note that party  $A$  accepts  $k_t$  if and only if  $k_t \leq 2\theta_{At}$ . Hence, if  $\theta_{Bt} \leq 2\theta_{At}$ , party  $B$ 's optimal proposal is  $k_t = \theta_{Bt}$ , and if  $\theta_{Bt} > 2\theta_{At}$ , party  $B$ 's optimal proposal is  $k_t = 2\theta_{At}$ . Therefore, we have the following result.

**Proposition 3.** *Under a budgetary institution that allows only discretionary spending, given the initial status quo of zero spending, the equilibrium is statically Pareto efficient and dynamically Pareto inefficient.*

Static efficiency obtains since the equilibrium spending is in the interval  $[\theta_{At}, \theta_{Bt}]$  for all  $t$ . Dynamic inefficiency arises because the equilibrium spending level depends on the identity of the proposer, hence there is dynamic inefficiency due to political risk.<sup>23</sup>

#### 4.2. Mandatory spending only

Suppose now that spending is allocated through mandatory programs only, that is,  $Z = \{0\} \times \mathbb{R}_+$ . Since  $k_t$  is zero for any  $t$ , for the rest of this subsection we denote a proposal in period  $t$  by  $g_t$ . Mandatory spending creates a dynamic link between periods because the first-period spending becomes the second-period status quo. We show that equilibrium allocations are in general dynamically Pareto inefficient and can even be statically Pareto inefficient. These results are in accordance with others that have shown inefficiency with an endogenous status quo and evolving preferences in settings different from ours.<sup>24</sup> By demonstrating inefficiency with mandatory spending only in our setting, we highlight its sources: political risk and gridlock.

**Proposition 4.** *Under a budgetary institution that allows only mandatory spending, an equilibrium exists, and for any equilibrium  $\sigma^*$ , there is at most one initial status quo  $g_0 \in \mathbb{R}_+$  such that  $\sigma^*$  is dynamically Pareto efficient given  $g_0$ .*

To gain some intuition for [Proposition 4](#), consider the equilibrium level of spending in the second period as a function of the second-period status quo  $g_1 \in \mathbb{R}_+$ . As illustrated in [Fig. 1](#), if the status quo is in  $[\theta_{A2}, \theta_{B2}]$ , then the period-2 spending is equal to the status quo, so there is gridlock. If the status quo is outside  $[\theta_{A2}, \theta_{B2}]$ , then the period-2 spending depends on the identity of the proposer and political risk is the source of dynamic inefficiency.

<sup>23</sup> Political risk is the only source of inefficiency in part because the status quo spending in the discretionary only institution is exogenously zero, and hence always lower than both parties' ideal points. If the exogenous status quo is between the ideal points in both periods, then the source of dynamic inefficiency is gridlock because in equilibrium the spending is stuck at the status quo. We find it natural that in our model of public spending the exogenous status quo is fixed at zero.

<sup>24</sup> For example, [Riboni and Ruge-Murcia \(2008\)](#) show dynamic inefficiency in the context of central bank decision-making, and [Zápal \(2011\)](#) and [Dziuda and Loeper \(2016\)](#) show static inefficiency in other settings.

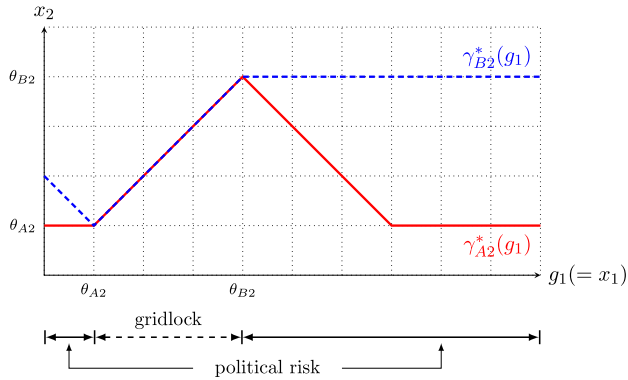


Fig. 1. Period-2 equilibrium strategies with only mandatory spending.

The next result shows that equilibrium allocations under mandatory spending programs can violate not only dynamic, but also static Pareto efficiency.

**Proposition 5.** *Under a budgetary institution that allows only mandatory spending, if either  $\theta_{A2} < \theta_{A1} < \theta_{B2}$  or  $\theta_{A2} < \theta_{B1} < \theta_{B2}$ , then there exists a nonempty open interval  $\mathcal{I}$  such that any equilibrium  $\sigma^*$  is statically Pareto inefficient for any initial status quo  $g_0 \in \mathcal{I}$ .*

The key condition of Proposition 5 is  $\theta_{A2} < \theta_{i1} < \theta_{B2}$  for some  $i \in \{A, B\}$ . This has a natural interpretation, indicating that future polarization between the two parties must be greater than intertemporal preference variation for at least one party.<sup>25</sup>

The reason for static inefficiency is the dual role of  $g_1$ : it is the spending in period 1 but it also determines the status quo in period 2. Suppose party A is the proposer in the first period and  $\theta_{A2} < \theta_{A1}$ . Then it has an incentive to propose spending close to  $\theta_{A1}$ , but since period-1 spending is the status quo for period 2, it also has an incentive to propose spending lower than  $\theta_{A1}$ . When party B’s acceptance constraint is not binding, party A proposes spending that is a weighted average of  $\theta_{A1}$  and  $\theta_{A2}$ , giving rise to static inefficiency.

### 5. Flexible budgetary institutions

We have seen that discretionary or mandatory programs in isolation typically lead to dynamic inefficiency. A natural question is which budgetary institutions achieve dynamic efficiency. We address this question first in the two-period model with deterministically evolving ideals and then in more complex environments. For this section we return to the general model without the functional form assumption on  $u_{it}$  and only assume that the parties’ ideals are strictly positive.

#### 5.1. Mandatory and discretionary combined

Suppose the parties can endogenously choose the amount allocated to mandatory and discretionary programs, that is,  $Z = \{(k_t, g_t) \in \mathbb{R} \times \mathbb{R}_+ | k_t + g_t \geq 0\}$ . We show that this budgetary

<sup>25</sup> In the working paper version (Bowen et al., 2016) we show that static Pareto inefficiency can arise even in the absence of first-period conflict between the two parties, that is, if  $\theta_{A1} = \theta_{B1}$ .

institution leads to dynamic Pareto efficiency in the baseline two-period model with deterministically evolving ideals.

**Proposition 6.** *Under a budgetary institution that allows both mandatory and discretionary spending, an equilibrium exists, and every equilibrium  $\sigma^*$  is dynamically Pareto efficient for any initial status quo  $g_0 \in \mathbb{R}_+$ .*

The reason the combination of mandatory and discretionary spending achieves dynamic Pareto efficiency is that the proposer in the first period can perfectly tailor the spending in that period to first-period preferences, and independently choose the next period's status quo. Specifically, note that if the status quo in the second period is between the ideals of the two parties, then, since it is statically Pareto efficient, it is maintained regardless of who the second-period proposer is. Thus, the first-period proposer *effectively* specifies the entire sequence of allocations by choosing a level of mandatory spending that is maintained in the second period, and combining it with discretionary spending to reflect the preferences in the first period. By doing this, the parties avoid both political risk and gridlock in equilibrium. Hence, combining discretionary and mandatory spending provides sufficient flexibility to achieve dynamic Pareto efficiency.<sup>26</sup>

When there are more than two periods or preferences evolve stochastically, however, simply combining mandatory and discretionary spending no longer allows the proposer to perfectly tailor the spending in the current period to the preferences in that period and independently choose the status quos for all future periods. Therefore efficiency can no longer be achieved. In order to achieve efficiency, more flexibility is needed. We illustrate next that if preferences evolve deterministically, then sunset provisions with appropriately chosen expiration dates achieve efficiency with more than two periods.

## 5.2. Sunset provisions

Consider the following three-period extension with sunset provisions. A proposal in period  $t$  is  $z_t = (k_t, s_t, g_t)$ . As before,  $k_t$  is discretionary spending for period  $t$  and  $g_t$  is mandatory spending. The new component is sunset provision  $s_t$ , which is spending that applies in periods  $t$  and  $t + 1$  and expires thereafter. If  $z_t$  is accepted, then the spending in period  $t$  is  $x_t = k_t + s_t + g_t$  and the status quo in period  $t + 1$  is  $(s_t, g_t)$ . If  $z_t$  is rejected, then the spending in period  $t$  is  $x_t = s_{t-1} + g_{t-1}$  and the status quo in period  $t + 1$  is  $(0, g_{t-1})$ .

Note that an accepted proposal  $z_1 = (k_1, s_1, g_1)$  determines spending in the first period  $x_1 = k_1 + s_1 + g_1$ , the status quo spending in the second period  $x_2 = s_1 + g_1$  and the status quo spending in the third period  $x_3 = g_1$ . Therefore, sunset provisions in combination with mandatory and discretionary spending allow the proposer to choose today's spending independently of future status quos, and choose future status quos independently of each other (this is not possible with only mandatory and discretionary spending). In the first period the proposer can tailor the status quo spending for each future period such that the future proposer in that period has no incentive to change it. Similar to the intuition for [Proposition 6](#), the equilibrium first-period

<sup>26</sup> [Proposition 6](#) allows both positive and negative discretionary spending. In the working paper version ([Bowen et al., 2016](#)) we also consider a budgetary institution that allows only positive discretionary and mandatory spending. In [Proposition 7](#) of that paper, we show that when the ideal values of the parties are decreasing and under certain regularity conditions on stage utilities, dynamic Pareto efficiency obtains even with this more restrictive budgetary institution.

proposal  $z_1 = (k_1, s_1, g_1)$  induces an allocation  $\mathbf{x} = (k_1 + s_1 + g_1, s_1 + g_1, g_1)$ , which is dynamically Pareto efficient and remains unchanged in later periods. This avoids gridlock by allowing spending to fluctuate with the evolving preferences, and eliminates political risk by ensuring that the spending levels do not depend on the identity of future proposers. This result holds more generally. Specifically, beyond three periods, multiple sunset provisions with different expiration dates allow the proposer to choose status quo spending for each future period independently and therefore provide sufficient flexibility required for dynamic efficiency.<sup>27</sup>

Stochastically evolving preferences require further flexibility for efficiency to be achieved. In the next section we consider a model with arbitrary time horizon and stochastic preferences and show that state-contingent mandatory spending provides such flexibility.

### 5.3. State-contingent mandatory spending

Consider a richer environment in which parties bargain in  $T \geq 2$  periods and preferences are stochastic in each period reflecting uncertainties in the economy.<sup>28</sup> The economic state (henceforth we refer to the economic state as simply the state) in each period  $t$  is  $s_t \in S$  where  $S$  is a finite set of  $n = |S|$  possible states. We assume the distribution of states has full support in every period, but do not require the distribution to be the same across periods.<sup>29</sup> The utility of party  $i$  in period  $t$  when the spending is  $x$  and the state is  $s$  is  $u_i(x, s)$ . As before, we assume  $u_i(x, s)$  is twice continuously differentiable and strictly concave in  $x$ . Further,  $u_i(x, s)$  attains a maximum at  $\theta_{is}$  and we assume  $\theta_{is} > 0$  for all  $i \in \{A, B\}$  and all  $s \in S$ . The state is drawn at the beginning of each period before a proposal is made. We denote the probability that party  $i$  proposes in period  $t$  by  $p_{it} \in (0, 1)$ , which can depend on  $t$  arbitrarily. In this setting, we consider a budgetary institution that allows state-contingent mandatory spending. As discussed in the Introduction, these state-contingent programs have been used historically, and are still in use.

A proposal in period  $t$  is a *spending rule*  $g_t : S \rightarrow \mathbb{R}_+$  where  $g_t(s)$  is the level of public good spending proposed to be allocated to the mandatory program in state  $s$ . If the responding party agrees to the proposal, the allocation implemented in period  $t$  is  $g_t(s_t)$ ; otherwise the allocation in period  $t$  is given by the status quo spending rule  $g_{t-1}(s_t)$ . In this environment, a strategy for party  $i$  in period  $t$  is  $\sigma_{it} = (\gamma_{it}, \alpha_{it})$ . Let  $\mathcal{M}$  be the space of all functions from  $S$  to  $\mathbb{R}_+$ . Then  $\gamma_{it} : \mathcal{M} \times S \rightarrow \mathcal{M}$  is a proposal strategy for party  $i$  in period  $t$  and  $\alpha_{it} : \mathcal{M} \times S \times \mathcal{M} \rightarrow \{0, 1\}$  is an acceptance strategy for party  $i$  in period  $t$ . A strategy for party  $i$  is  $\sigma_i = (\sigma_{i1}, \dots, \sigma_{iT})$  and a profile of strategies is  $\sigma = (\sigma_1, \sigma_2)$ .

With stochastic preferences the social planner chooses a spending rule  $x_t : S \rightarrow \mathbb{R}_+$  for all  $t \in \{1, \dots, T\}$  to maximize the expected payoff of one of the parties subject to providing the other party with a minimum expected dynamic payoff. Formally, a dynamically Pareto efficient allocation rule solves the following maximization problem:

<sup>27</sup> Note that with sunset provisions, flexibility is introduced by adding dimensions to the policy space. It is possible that these additional policy instruments are not available, in which case one might ask if *reducing* flexibility by placing bounds on mandatory and discretionary programs might improve efficiency. That is, we might restrict the set of policies to  $Z = [a, b] \times [c, d] \subseteq \mathbb{R} \times \mathbb{R}_+$  and ask what values of  $(a, b, c, d)$  are optimal. We leave this inquiry to future work.

<sup>28</sup> Note that  $T$  can be finite or infinite. In the case of infinite horizon we assume  $\delta \in (0, 1)$  so that dynamic utilities are well-defined.

<sup>29</sup> We assume full support in this section for expositional simplicity, but Proposition 8 below still holds in an extension in which the distribution of states has different supports in different periods.

$$\begin{aligned} \max_{\{x_t: S \rightarrow \mathbb{R}_+\}_{t=1}^T} \quad & \sum_{t=1}^T \delta^{t-1} \mathbb{E}_{s_t} [u_i(x_t(s_t), s_t)] \\ \text{s.t.} \quad & \sum_{t=1}^T \delta^{t-1} \mathbb{E}_{s_t} [u_j(x_t(s_t), s_t)] \geq \bar{U}, \end{aligned} \tag{DSP-S}$$

for some  $\bar{U} \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ . We denote the solution to (DSP-S) by the sequence of functions  $\mathbf{x}^* = \{x_t^*\}_{t=1}^T$ . The next proposition characterizes dynamically Pareto efficient allocation rules, analogous to Proposition 2.

**Proposition 7.** Any dynamically Pareto efficient allocation rule satisfies:

1. For any  $t$  and  $t'$ ,  $x_t^* = x_{t'}^*$ .
2. For all  $s \in S$  and all  $t$ , either

$$-\frac{u'_i(x_t^*(s), s)}{u'_j(x_t^*(s), s)} = \lambda^*$$

for some  $\lambda^* > 0$ , or  $x_t^*(s) = \theta_{As}$ , or  $x_t^*(s) = \theta_{Bs}$ .

Proposition 7 first says that the dynamically Pareto efficient allocation rule is independent of time, i.e., the same spending rule is used each period. The second part of the proposition says that the dynamically Pareto efficient allocation rule either satisfies the condition that the ratio of the parties' marginal utilities is constant across states, or is one party's ideal in each state.

We next define a dynamically Pareto efficient equilibrium and show that dynamic efficiency is obtained by state-contingent mandatory spending. Define recursively  $x_t^{\sigma^*}(g_0)$  for  $t \in \{1, \dots, T\}$  by  $x_1^{\sigma^*}(g_0) = \gamma_{i1}^*(g_0, s_1)$  for some  $i \in \{A, B\}$  and some  $s_1 \in S$ , and  $x_t^{\sigma^*}(g_0) = \gamma_{it}^*(x_{t-1}^{\sigma^*}(g_0), s_t)$  for some  $i \in \{A, B\}$  and some  $s_t \in S$  and  $t \in \{2, \dots, T\}$ . An equilibrium allocation rule for  $\sigma^*$  given initial status quo  $g_0$  is a possible realization of a spending rule for each period,  $\mathbf{x}^{\sigma^*}(g_0) = \{x_t^{\sigma^*}(g_0)\}_{t=1}^T$ . The random determination of proposers and states in each period induces a probability distribution over allocation rules given an equilibrium  $\sigma^*$ . Thus any element in the support of this distribution is an equilibrium allocation rule for  $\sigma^*$ .

**Definition 2.** An equilibrium  $\sigma^*$  is a dynamically Pareto efficient equilibrium given initial status quo  $g_0 \in \mathcal{M}$  if and only if

1. every equilibrium allocation rule  $\mathbf{x}^{\sigma^*}(g_0)$  is dynamically Pareto efficient; and
2. given the first-period proposer, every equilibrium allocation rule  $\mathbf{x}^{\sigma^*}(g_0)$  is identical.

**Proposition 8.** Under state-contingent mandatory spending, an equilibrium is either dynamically Pareto efficient for any initial status quo  $g_0 \in \mathcal{M}$  or is outcome-equivalent to a dynamically Pareto efficient equilibrium.

The result in Proposition 8 is in stark contrast to the inefficiency results for mandatory spending given in Propositions 4 and 5. Recall that dynamic efficiency fails in the model with evolving (deterministic) preferences and fixed mandatory spending because the proposer in period 1 cannot specify spending in the current period separately from the status quo for the next period. Proposition 8 can be understood in an analogous way to the efficiency result with discretionary and mandatory spending combined. In the first period the proposer can tailor the status quo for each state such that any acceptable proposal other than the status quo makes the future proposer

worse off, giving the future proposer no incentive to change it. This avoids gridlock by allowing spending to fluctuate with the economic state, and eliminates political risk by ensuring that the spending levels do not depend on the identity of future proposers.<sup>30</sup> Thus, even though the identity of the proposer is not contractible, in equilibrium inefficiency arising from proposer uncertainty is eliminated through the status quo. State-contingent mandatory spending therefore overcomes inefficiency due to two kinds of uncertainty—uncertainty about states, which is contractible, and uncertainty about the proposer identity, which is not contractible.

## 6. Conclusion

In this paper we demonstrate that discretionary only and mandatory only budgetary institutions typically result in dynamic inefficiency, and may result in static inefficiency in the case of mandatory only budgetary institutions. However, we show that bargaining achieves dynamic Pareto efficiency in increasingly complex environments when flexibility is introduced through either an endogenous combination of mandatory and discretionary programs, sunset provisions, or a state-contingent mandatory program. We show that these budgetary institutions eliminate political risk and gridlock by allowing the proposer to choose status quos that are not changed by future proposers because they fully account for fluctuations in preferences.

We have considered mandatory spending programs that are fully state-contingent, but it is possible that factors influencing parties' preferences, such as the mood of the electorate, cannot be contracted on. In this case it seems there is room for inefficiency even with mandatory spending that depends on a contractible state. It is possible that further flexibility with discretionary spending may be helpful. Such combinations are observed in practice; for example, in the United States, unemployment insurance is provided through both state-contingent mandatory programs and discretionary programs.<sup>31</sup> However, including discretionary spending may leave more room for political risk. We leave for future work exploring efficiency implications of discretionary and mandatory spending when a part of the state may not be contracted on.

## Appendix A. Pareto efficiency

### A.1. Proof of Proposition 1

First, we show that if  $x_t$  is statically Pareto efficient, then  $x_t \in [\underline{\theta}_t, \bar{\theta}_t]$ . Consider  $x_t \notin [\underline{\theta}_t, \bar{\theta}_t]$ . Then we can find  $x'_t$  in either  $(x_t, \underline{\theta}_t)$  or  $(\bar{\theta}_t, x_t)$  such that  $u_{At}(x'_t) > u_{At}(x_t)$  and  $u_{Bt}(x'_t) > u_{Bt}(x_t)$ , and therefore  $x_t$  is not a solution to (SSP).

Second, we show that if  $\tilde{x}_t \in [\underline{\theta}_t, \bar{\theta}_t]$ , then  $\tilde{x}_t$  is statically Pareto efficient. If  $\underline{\theta}_t = \bar{\theta}_t$ , the claim is obvious, so suppose  $\underline{\theta}_t < \bar{\theta}_t$ . Suppose  $\theta_{At} < \theta_{Bt}$ . When  $\theta_{At} > \theta_{Bt}$  the argument is similar and omitted. Let  $\bar{u} = u_{jt}(\tilde{x}_t)$ . Denote the solution to (SSP) as  $\hat{x}_t(\bar{u})$ . Since  $u'_{At}(x_t) < 0$  and  $u'_{Bt}(x_t) > 0$  for all  $x_t \in (\theta_{At}, \theta_{Bt})$ , the solution to (SSP) is  $\hat{x}_t(\bar{u}) = \tilde{x}_t$  for any  $\tilde{x}_t \in [\theta_{At}, \theta_{Bt}]$ .  $\square$

<sup>30</sup> We can regard the two-period model analyzed previously as a special case of an extension of this subsection's model in which the distribution of states has different supports in different periods with a degenerate distribution in each period. The state-contingent mandatory programs achieve dynamic efficiency by allowing the total spending in period 1 to be different from the status quo spending in period 2, which can also be achieved by a combination of mandatory and discretionary programs in the two-period setting.

<sup>31</sup> See *Department of Labor Budget in Brief*, United States Department of Labor (2015).



A.2. Proof of Proposition 2

We prove the result for a more general model with  $T \geq 2$ . Denote a sequence of allocations by  $\mathbf{x} = \{x_t\}_{t=1}^T$  and party  $i$ 's discounted dynamic payoff from  $\mathbf{x} = \{x_t\}_{t=1}^T$  by  $U_i(\mathbf{x}) = \sum_{t=1}^T \delta^{t-1} u_{it}(x_t)$ . We define a dynamically Pareto efficient allocation in the  $T$ -period problem as the solution to the following maximization problem

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}_+^T} U_i(\mathbf{x}) \\ \text{s.t. } U_j(\mathbf{x}) \geq \bar{U} \end{aligned} \tag{DSP-T}$$

for some  $\bar{U} \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ . Denote the sequence of party  $i$ 's static ideals by  $\theta_i = \{\theta_{it}\}_{t=1}^T$  for all  $i \in \{A, B\}$ , and denote the solution to (DSP-T) as  $\mathbf{x}^* = \{x_t^*\}_{t=1}^T$ .

To prove part 1, by way of contradiction, suppose  $x_{t'}^* \notin [\underline{\theta}_{t'}, \bar{\theta}_{t'}]$ . By Proposition 1 there exists  $\hat{x}_{t'}$  such that  $u_{it'}(\hat{x}_{t'}) \geq u_{it'}(x_{t'}^*)$  for all  $i \in \{A, B\}$ , and  $u_{it'}(\hat{x}_{t'}) > u_{it'}(x_{t'}^*)$  for at least one  $i \in \{A, B\}$ . Now consider  $\hat{\mathbf{x}} = \{\hat{x}_t\}_{t=1}^T$ , with  $\hat{x}_t = x_t^*$  for all  $t \neq t'$ . Then  $U_i(\hat{\mathbf{x}}) \geq U_i(\mathbf{x}^*)$  for all  $i \in \{A, B\}$ , and  $U_i(\hat{\mathbf{x}}) > U_i(\mathbf{x}^*)$  for at least one  $i \in \{A, B\}$ . Thus  $\mathbf{x}^*$  is not dynamically Pareto efficient.

Next we prove part 2. Since  $u_{it}$  is concave for all  $i$  and  $t$ , the utility possibility set  $\{(U_A, U_B) \in \mathbb{R}_+^2 \mid \exists \mathbf{x} \in \mathbb{R}_+^T \text{ s.t. } U_i \leq U_i(\mathbf{x}) \text{ for all } i \in \{A, B\}\}$  is convex, and hence by Mas-Colell et al. (1995, Proposition 16.E.2),  $\mathbf{x}^*$  is a solution to

$$\max_{\mathbf{x} \in \mathbb{R}_+^T} \lambda_A U_A(\mathbf{x}) + \lambda_B U_B(\mathbf{x}) \tag{DSP-λ}$$

for some  $(\lambda_A, \lambda_B) \in \mathbb{R}_+^2 \setminus (0, 0)$ .

By part 1,  $x_t^* \in [\underline{\theta}_t, \bar{\theta}_t] \subset \mathbb{R}_+$  for all  $t$ . And therefore,  $x_t^*$  satisfies the following first order condition

$$\lambda_A u'_{At}(x_t^*) + \lambda_B u'_{Bt}(x_t^*) = 0 \tag{A1}$$

for all  $t$ . If  $\lambda_B = 0$ , then  $\mathbf{x}^* = \theta_A$ . If  $\lambda_A = 0$ , then  $\mathbf{x}^* = \theta_B$ . If  $\lambda_A, \lambda_B > 0$ , dividing (A1) by  $\lambda_A$  and denoting  $\lambda^* = \frac{\lambda_B}{\lambda_A} > 0$  gives  $u'_{At}(x_t^*) + \lambda^* u'_{Bt}(x_t^*) = 0$  for all  $t$ . □

Appendix B. Rigid budgetary institutions

B.1. Proof of Lemma 1

We prove Lemma 1 by using Lemma A1, which we state and prove below.

**Lemma A1.** Suppose  $u_{it}(x_t) = -(x_t - \theta_{it})^2$  for all  $i \in \{A, B\}$  and  $t$ . Then an allocation  $\mathbf{x}$  is dynamically Pareto efficient if and only if  $\mathbf{x} = \alpha \theta_A + (1 - \alpha) \theta_B$  for some  $\alpha \in [0, 1]$ .

**Proof.** We first show the ‘if’ part. For any  $(\lambda_A, \lambda_B) \in \mathbb{R}_{++}^2$ , if  $\hat{\mathbf{x}}$  solves (DSP-λ) with  $(\lambda_A, \lambda_B)$ , then  $\hat{\mathbf{x}}$  is dynamically Pareto efficient by Mas-Colell et al. (1995, Proposition 16.E.2). From the proof of Proposition 2 part 2, the condition  $\lambda_A u'_{At}(\hat{x}_t) + \lambda_B u'_{Bt}(\hat{x}_t) = 0$  for all  $t$  is necessary for  $\hat{\mathbf{x}}$  to be a solution to (DSP-λ) with  $(\lambda_A, \lambda_B)$ . Sufficiency follows from the concavity of  $u_{it}$  for all  $i$  and  $t$ . For the quadratic utility functions this condition can be rewritten as  $\hat{x}_t = \frac{\lambda_A}{\lambda_A + \lambda_B} \theta_{At} + \frac{\lambda_B}{\lambda_A + \lambda_B} \theta_{Bt}$  for all  $t$ . Hence,  $\hat{\mathbf{x}} = \alpha \theta_A + (1 - \alpha) \theta_B$  where  $\alpha = \frac{\lambda_A}{\lambda_A + \lambda_B}$  is a dynamically Pareto efficient allocation. Note that for any  $\alpha \in (0, 1)$  there exists  $(\lambda_A, \lambda_B) \in \mathbb{R}_{++}^2$

such that  $\alpha = \frac{\lambda_A}{\lambda_A + \lambda_B}$ . Next consider  $\alpha \in \{0, 1\}$ . Fixing  $i = A$  and  $j = B$  in (DSP-T),  $\hat{x} = \theta_B$  is dynamically Pareto efficient, as it solves (DSP-T) with  $\bar{U} = U_B(\theta_B)$ , and  $\hat{x} = \theta_A$  is dynamically Pareto efficient, as it solves (DSP-T) with any  $\bar{U} \leq U_B(\theta_A)$ .

We next show the ‘only if’ part. As shown in the proof of Proposition 2 part 2, any dynamically Pareto efficient allocation  $x^*$  solves (DSP- $\lambda$ ) for some  $(\lambda_A, \lambda_B) \in \mathbb{R}_+^2 \setminus (0, 0)$ . Thus  $x^* = \alpha\theta_A + (1 - \alpha)\theta_B$ , where  $\alpha = \frac{\lambda_A}{\lambda_A + \lambda_B} \in [0, 1]$ .  $\square$

From Lemma A1, there is a continuum of dynamically Pareto efficient allocations since  $\theta_A \neq \theta_B$ . Moreover, if there exists  $x^*$  with  $x_1^* = x_2^*$ , then it satisfies  $\alpha^*\theta_{A1} + (1 - \alpha^*)\theta_{B1} = \alpha^*\theta_{A2} + (1 - \alpha^*)\theta_{B2}$  for some  $\alpha^* \in [0, 1]$ . This requires that  $\alpha^*[\theta_{B1} - \theta_{B2} - (\theta_{A1} - \theta_{A2})] = \theta_{B1} - \theta_{B2}$ . If  $\theta_{B1} - \theta_{B2} - (\theta_{A1} - \theta_{A2}) = 0$ , this cannot be satisfied since  $\theta_{B1} \neq \theta_{B2}$ . If  $\theta_{B1} - \theta_{B2} - (\theta_{A1} - \theta_{A2}) \neq 0$ , then  $\alpha^* = \frac{\theta_{B1} - \theta_{B2}}{\theta_{B1} - \theta_{B2} - (\theta_{A1} - \theta_{A2})}$ . Hence, there exists at most one  $x^*$  with  $x_1^* = x_2^*$ .  $\square$

### B.2. Proof of Proposition 4

Equilibrium existence follows from Proposition A1 in Appendix C. To prove the remainder of the proposition, we use the following three lemmas.

**Lemma A2.** Let  $Z = \{0\} \times \mathbb{R}_+$ . Suppose  $u_{it}(x_t) = -(x_t - \theta_{it})^2$  for all  $i \in \{A, B\}$  and  $t$  and  $\theta_{A2} < \theta_{B2}$ . Then

$$\begin{aligned} \gamma_{A2}^*(g_1) &= \max\{\theta_{A2}, \min\{g_1, 2\theta_{B2} - g_1\}\}, \\ \gamma_{B2}^*(g_1) &= \min\{\theta_{B2}, \max\{g_1, 2\theta_{A2} - g_1\}\}. \end{aligned}$$

Hence, we have

1.  $\gamma_{i2}^*(g_1) \in [\theta_{A2}, \theta_{B2}]$  for all  $i \in \{A, B\}$  and  $g_1 \in \mathbb{R}_+$ ;
2.  $\gamma_{A2}^*(g_1) = \gamma_{B2}^*(g_1) = g_1$  if  $g_1 \in [\theta_{A2}, \theta_{B2}]$  and  $\gamma_{A2}^*(g_1) \neq \gamma_{B2}^*(g_1)$  if  $g_1 \notin [\theta_{A2}, \theta_{B2}]$ .

**Proof.** We prove that  $\gamma_{A2}^*(g_1) = \max\{\theta_{A2}, \min\{g_1, 2\theta_{B2} - g_1\}\}$ . There are two possible cases.

**Case (i):**  $|g_1 - \theta_{B2}| \geq |\theta_{A2} - \theta_{B2}|$ . Note that  $|g_1 - \theta_{B2}| \geq |\theta_{A2} - \theta_{B2}|$  is equivalent to  $u_{B2}(g_1) \leq u_{B2}(\theta_{A2})$ . Hence party B accepts  $g_2 = \theta_{A2}$  and, since  $\theta_{A2}$  is the unique maximizer of  $u_{A2}$ ,  $\gamma_{A2}^*(g_1) = \theta_{A2}$ . Note also that  $\theta_{A2} \geq \min\{g_1, 2\theta_{B2} - g_1\}$  when  $|g_1 - \theta_{B2}| \geq |\theta_{A2} - \theta_{B2}| \Leftrightarrow g_1 \notin (\theta_{A2}, 2\theta_{B2} - \theta_{A2})$ .

**Case (ii):**  $|g_1 - \theta_{B2}| < |\theta_{A2} - \theta_{B2}|$ . In this case party B accepts  $g_2$  if and only if  $|g_2 - \theta_{B2}| \leq |g_1 - \theta_{B2}|$ , or, equivalently, if and only if  $g_2 \in [\min\{g_1, 2\theta_{B2} - g_1\}, \max\{g_1, 2\theta_{B2} - g_1\}]$ . Since  $\theta_{A2} < \min\{g_1, 2\theta_{B2} - g_1\}$  when  $|g_1 - \theta_{B2}| < |\theta_{A2} - \theta_{B2}| \Leftrightarrow g_1 \in (\theta_{A2}, 2\theta_{B2} - \theta_{A2})$ ,  $u_{A2}$  is strictly decreasing on  $[\min\{g_1, 2\theta_{B2} - g_1\}, \max\{g_1, 2\theta_{B2} - g_1\}]$  and thus  $\gamma_{A2}^*(g_1) = \min\{g_1, 2\theta_{B2} - g_1\}$ .

The proof that  $\gamma_{B2}^*(g_1) = \min\{\theta_{B2}, \max\{g_1, 2\theta_{A2} - g_1\}\}$  is analogous and omitted. Parts 1 and 2 follow immediately.  $\square$

**Lemma A3.** Let  $Z = \{0\} \times \mathbb{R}_+$ . Suppose  $u_{it}(x_t) = -(x_t - \theta_{it})^2$  for all  $i \in \{A, B\}$  and  $t$  and  $\theta_{A2} < \theta_{B2}$ . For any  $g_0 \in \mathbb{R}_+$ , if  $\sigma^*$  is a dynamically Pareto efficient equilibrium given  $g_0$ , then any equilibrium allocation  $x^{\sigma^*}(g_0)$  satisfies  $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ .

**Proof.** Fix a dynamically Pareto efficient equilibrium  $\sigma^*$  given  $g_0$ . The equilibrium spending in period 2 is either  $\gamma_{A2}^*(x_1^{\sigma^*}(g_0))$  or  $\gamma_{B2}^*(x_1^{\sigma^*}(g_0))$ . Since  $\sigma^*$  is an equilibrium, we have either  $\gamma_{A2}^*(x_1^{\sigma^*}(g_0)) \neq \gamma_{B2}^*(x_1^{\sigma^*}(g_0))$  or  $x_2^{\sigma^*}(g_0) = \gamma_{A2}^*(x_1^{\sigma^*}(g_0)) = \gamma_{B2}^*(x_1^{\sigma^*}(g_0)) = x_1^{\sigma^*}(g_0)$  by Lemma A2 part 2. In the former case, the level of spending in period 2 depends on the identity of the period-2 proposer, contradicting that  $\sigma^*$  is a dynamically Pareto efficient equilibrium given  $g_0$ . Thus, we must have  $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ .  $\square$

**Lemma A4.** Suppose  $u_{ii}(x_i) = -(x_i - \theta_{ii})^2$  for all  $i \in \{A, B\}$  and  $t$  and  $\theta_{i1} \neq \theta_{i2}$  for all  $i \in \{A, B\}$ . Then  $\mathbf{x}^*$  with  $x_1^* = x_2^*$  exists if and only if  $\text{sgn}[\theta_{B1} - \theta_{B2}] = \text{sgn}[\theta_{A2} - \theta_{A1}]$ . If  $\mathbf{x}^*$  with  $x_1^* = x_2^*$  exists, then  $\mathbf{x}^* = \mathbf{x}^*(\alpha^*) = \alpha^* \boldsymbol{\theta}_A + (1 - \alpha^*) \boldsymbol{\theta}_B$ , where  $\alpha^* = \frac{\theta_{B1} - \theta_{B2}}{\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1}}$ .

**Proof.** From Lemma A1,  $\mathbf{x}^*$  with  $x_1^* = x_2^*$  exists if and only if there exists  $\alpha^* \in [0, 1]$  such that  $\alpha^* \theta_{A1} + (1 - \alpha^*) \theta_{B1} = \alpha^* \theta_{A2} + (1 - \alpha^*) \theta_{B2}$ , or, equivalently,  $\alpha^* [\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1}] = \theta_{B1} - \theta_{B2}$ . When  $\text{sgn}[\theta_{B1} - \theta_{B2}] = \text{sgn}[\theta_{A2} - \theta_{A1}]$ , we have  $\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1} \neq 0$  and  $\alpha^* = \frac{\theta_{B1} - \theta_{B2}}{\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1}} \in (0, 1)$ . When  $\text{sgn}[\theta_{B1} - \theta_{B2}] \neq \text{sgn}[\theta_{A2} - \theta_{A1}]$ , either  $\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1} = 0$  and  $\alpha^* [\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1}] = \theta_{B1} - \theta_{B2}$  cannot be satisfied since  $\theta_{B1} \neq \theta_{B2}$ , or  $\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1} \neq 0$  and  $\alpha^* = \frac{\theta_{B1} - \theta_{B2}}{\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1}} \notin [0, 1]$ .  $\square$

Suppose there exists  $\sigma^*$  such that  $\sigma^*$  is a dynamically Pareto efficient equilibrium given some  $g_0$ . We next show that there is at most one such  $g_0$ . Fix  $\sigma^*$  and  $g_0$  such that  $\sigma^*$  is a dynamically Pareto efficient equilibrium given  $g_0$ .

By Lemma A3, we have  $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ . If  $\text{sgn}[\theta_{B1} - \theta_{B2}] \neq \text{sgn}[\theta_{A2} - \theta_{A1}]$ , then  $(x_1^{\sigma^*}(g_0), x_2^{\sigma^*}(g_0))$  is not a dynamically Pareto efficient allocation since any dynamically Pareto efficient allocation satisfies  $x_1^* \neq x_2^*$  by Lemma A4. Hence, we must have  $\text{sgn}[\theta_{B1} - \theta_{B2}] = \text{sgn}[\theta_{A2} - \theta_{A1}]$ , and by Lemma A4, we have  $x_1^{\sigma^*}(g_0) = x_1^*(\alpha^*)$  and  $x_2^{\sigma^*}(g_0) = x_2^*(\alpha^*)$ . Since  $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ , it follows that  $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0) = x_1^*(\alpha^*) = x_2^*(\alpha^*)$ .

The remainder of the proof consists of three steps. To facilitate the exposition, let  $V_i(g_1) = p_A u_{i2}(\gamma_{A2}^*(g_1)) + p_B u_{i2}(\gamma_{B2}^*(g_1))$  be the expected second-period equilibrium payoff of party  $i \in \{A, B\}$  given  $g_1 \in \mathbb{R}_+$  and let  $f_i(g_1) = u_{i1}(g_1) + \delta V_i(g_1)$  be its dynamic (expected) utility from  $g_1 \in \mathbb{R}_+$ . From Lemma A10 in Appendix C,  $V_i$  and  $f_i$  are both continuous for all  $i \in \{A, B\}$ .

**Step 1:** We show that  $\gamma_{A1}^*(g_0) = \gamma_{B1}^*(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$ . We know that  $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0) = x_1^*(\alpha^*) = x_2^*(\alpha^*)$ . Since  $x_1^{\sigma^*}(g_0) = x_1^*(\alpha^*)$  where  $\alpha^*$  is unique, we have  $x_1^{\sigma^*}(g_0) = \gamma_{A1}^*(g_0) = \gamma_{B1}^*(g_0)$ . Moreover, we have  $x_1^*(\alpha^*) \in (\theta_{A1}, \theta_{B1})$  and  $x_2^*(\alpha^*) \in (\theta_{A2}, \theta_{B2})$  since  $\alpha^* \in (0, 1)$  when  $\text{sgn}[\theta_{B1} - \theta_{B2}] = \text{sgn}[\theta_{A2} - \theta_{A1}] \neq 0$ . Hence,  $x_1^{\sigma^*}(g_0) = x_1^*(\alpha^*) = x_2^*(\alpha^*)$  implies  $x_1^{\sigma^*}(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$  and thus  $\gamma_{A1}^*(g_0) = \gamma_{B1}^*(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$ .

**Step 2:** We now show that  $f_A(\gamma_{A1}^*(g_0)) = f_A(g_0)$  and  $f_B(\gamma_{A1}^*(g_0)) = f_B(g_0)$ . To see this, note that  $f_A(\gamma_{A1}^*(g_0)) \geq f_A(g_0)$  and  $f_B(\gamma_{A1}^*(g_0)) \geq f_B(g_0)$  since  $\gamma_{A1}^*(g_0)$  is proposed by A and accepted by B under status quo  $g_0$ . Suppose, towards a contradiction, that  $f_A(\gamma_{A1}^*(g_0)) > f_A(g_0)$  and  $f_B(\gamma_{A1}^*(g_0)) > f_B(g_0)$ . Since  $f_i(g_1) = u_{i1}(g_1) + \delta V_i(g_1)$ , where  $V_i(g_1) = u_{i2}(g_1)$  for all  $g_1 \in [\theta_{A2}, \theta_{B2}]$  by Lemma A2 part 2,  $f_A(g_1)$  is strictly decreasing and  $f_B(g_1)$  is strictly increasing in  $g_1$  on  $[\theta_{A1}, \theta_{B1}] \cap [\theta_{A2}, \theta_{B2}]$ . Since  $\gamma_{A1}^*(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$  and  $f_i$  is continuous, there exists  $\epsilon > 0$  such that  $f_A(\gamma_{A1}^*(g_0) - \epsilon) > f_A(\gamma_{A1}^*(g_0))$  and  $f_B(\gamma_{A1}^*(g_0) - \epsilon) > f_B(g_0)$ , so that proposing  $\gamma_{A1}^*(g_0)$  cannot be optimal for A, a contradiction. By a similar argument, it is impossible to have  $f_A(\gamma_{A1}^*(g_0)) = f_A(\gamma_{B1}^*(g_0)) > f_A(g_0)$  and  $f_B(\gamma_{A1}^*(g_0)) = f_B(\gamma_{B1}^*(g_0)) \geq f_B(g_0)$ .

**Step 3:** We show that if  $f_A(\gamma_{A1}^*(g_0)) = f_A(g_0)$  and  $f_B(\gamma_{A1}^*(g_0)) = f_B(g_0)$ , then  $g_0$  must be equal to  $x_1^*(\alpha^*)$ . As shown earlier,  $\gamma_{A1}^*(g_0) = x_1^*(\alpha^*)$ . We show that the system of equations

$$f_A(x_1^*(\alpha^*)) = f_A(g_0) \quad f_B(x_1^*(\alpha^*)) = f_B(g_0) \tag{A2}$$

has a unique solution in  $g_0$ . Clearly,  $g_0 = x_1^*(\alpha^*)$  solves (A2). To see that no other solution exists, suppose  $g' \neq x_1^*(\alpha^*)$  solves (A2). Since  $x_1^*(\alpha^*) \in [\theta_{A1}, \theta_{B1}] \cap [\theta_{A2}, \theta_{B2}]$  solves (A2) and  $f_A$  is strictly monotone on  $[\theta_{A1}, \theta_{B1}] \cap [\theta_{A2}, \theta_{B2}]$ , we must have  $g' \notin [\theta_{A1}, \theta_{B1}] \cap [\theta_{A2}, \theta_{B2}]$ .

Next we show that it is not possible to have  $g' \in \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$  using the following lemma.

**Lemma A5.** Let  $Z = \{0\} \times \mathbb{R}_+$ . Suppose  $u_{ii}(x_i) = -(x_i - \theta_{ii})^2$  for all  $i \in \{A, B\}$  and  $t$  and  $\theta_{A2} < \theta_{B2}$ . For any  $g_1 \in \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$ ,  $\tilde{g}(g_1) = p_A \gamma_{A2}^*(g_1) + p_B \gamma_{B2}^*(g_1)$  satisfies  $\tilde{g}(g_1) \in (\theta_{A2}, \gamma_{B2}^*(g_1)) \subseteq [\theta_{A2}, \theta_{B2}]$ ,  $V_A(g_1) < V_A(\tilde{g}(g_1))$  and  $V_B(g_1) < V_B(\tilde{g}(g_1))$ .

**Proof.** Fix  $g_1 < \theta_{A2}$ . From Lemma A2, if  $g_1 < \theta_{A2}$ , then  $\gamma_{A2}^*(g_1) = \theta_{A2}$  and  $\gamma_{B2}^*(g_1) > \theta_{A2}$ . We also have  $\gamma_{B2}^*(g_1) \leq \theta_{B2}$  by Lemma A2 part 1. Hence  $\tilde{g}(g_1) \in (\theta_{A2}, \gamma_{B2}^*(g_1)) \subseteq [\theta_{A2}, \theta_{B2}]$ .

Note that  $V_i(g_1) = p_A u_{i2}(\theta_{A2}) + p_B u_{i2}(\gamma_{B2}^*(g_1))$ . Moreover, since  $\tilde{g}(g_1) \in [\theta_{A2}, \theta_{B2}]$ , we have  $V_i(\tilde{g}(g_1)) = u_{i2}(\tilde{g}(g_1)) = u_{i2}(p_A \theta_{A2} + p_B \gamma_{B2}^*(g_1))$  by Lemma A2 part 2. By strict concavity of  $u_{i2}$ , it follows that  $V_i(g_1) < V_i(\tilde{g}(g_1))$  for  $i \in \{A, B\}$ . When  $g_1 > \theta_{B2}$ , the argument is similar and omitted.  $\square$

If  $g' \in \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$  solves (A2) we have  $f_i(g') = u_{i1}(g') + \delta V_i(g') = f_i(x_1^*(\alpha^*))$  for all  $i \in \{A, B\}$ . From Lemma A5, there exists  $\tilde{g}' \in [\theta_{A2}, \theta_{B2}]$  such that  $V_i(g') < V_i(\tilde{g}')$  for all  $i \in \{A, B\}$ . Since  $\tilde{g}' \in [\theta_{A2}, \theta_{B2}]$ ,  $V_i(\tilde{g}') = u_{i2}(\tilde{g}')$  by Lemma A2 part 2. Hence  $u_{i1}(g') + \delta u_{i2}(\tilde{g}') > f_i(x_1^*(\alpha^*))$  for all  $i \in \{A, B\}$ . Furthermore,  $f_i(x_1^*(\alpha^*)) = u_{i1}(x_1^*(\alpha^*)) + \delta u_{i2}(x_2^*(\alpha^*))$  for all  $i \in \{A, B\}$  since  $x_1^*(\alpha^*) = x_2^*(\alpha^*) \in [\theta_{A2}, \theta_{B2}]$ . Thus  $u_{i1}(g') + \delta u_{i2}(\tilde{g}') > u_{i1}(x_1^*(\alpha^*)) + \delta u_{i2}(x_2^*(\alpha^*))$  for all  $i \in \{A, B\}$ , which violates dynamic Pareto efficiency of  $(x_1^*(\alpha^*), x_2^*(\alpha^*))$  as it is Pareto dominated by  $(g', \tilde{g}')$ . Hence, it is not possible to have  $g' \in \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$ .

Since  $g' \notin [\theta_{A1}, \theta_{B1}] \cap [\theta_{A2}, \theta_{B2}]$  and  $g' \notin \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$ , it follows that if  $[\theta_{A2}, \theta_{B2}] \subseteq [\theta_{A1}, \theta_{B1}]$ , then  $g_0 = x_1^*(\alpha^*)$  is the unique solution to (A2). If instead  $[\theta_{A1}, \theta_{B1}] \subseteq [\theta_{A2}, \theta_{B2}]$ , then we need to rule out  $g' \in [\theta_{A2}, \theta_{A1}] \cup [\theta_{B1}, \theta_{B2}]$ . Note that  $f_B(g_0) = u_{B1}(g_0) + \delta u_{B2}(g_0)$  if  $g_0 \in [\theta_{A2}, \theta_{B1}]$  by Lemma A2 part 2, which implies that  $f_B(g_0)$  is strictly increasing in  $g_0$  on  $[\theta_{A2}, \theta_{B1}]$ . Since  $x_1^*(\alpha^*)$  solves (A2), it is not possible to have  $g' \in [\theta_{A2}, \theta_{A1}]$ . By a similar argument, it is not possible to have  $g' \in [\theta_{B1}, \theta_{B2}]$ . Thus, there is a unique solution to (A2) if  $[\theta_{A1}, \theta_{B1}] \subseteq [\theta_{A2}, \theta_{B2}]$ .  $\square$

**B.3. Proof of Proposition 5**

Let  $f_i$  and  $V_i$  be defined as in the proof of Proposition 4. Denote  $g_i^* \in \arg \max_{g_i \in \mathbb{R}_+} f_i(g_i)$  for  $i \in \{A, B\}$ . We first show that  $g_A^*$  is unique and  $g_A^* \notin [\theta_{A1}, \theta_{B1}]$  when  $\theta_{A2} < \theta_{A1} < \theta_{B2}$ . The proof that  $g_B^*$  is unique and  $g_B^* \notin [\theta_{A1}, \theta_{B1}]$  when  $\theta_{A2} < \theta_{B1} < \theta_{B2}$  is analogous and omitted.

Define  $Q_k \subset \mathbb{R}_+$  for  $k = 1, \dots, 5$  as

- $Q_1 = (0, \max \{0, 2\theta_{A2} - \theta_{B2}\})$ ,
- $Q_2 = (\max \{0, 2\theta_{A2} - \theta_{B2}\}, \theta_{A2})$ ,
- $Q_3 = (\theta_{A2}, \theta_{B2})$ ,
- $Q_4 = (\theta_{B2}, 2\theta_{B2} - \theta_{A2})$ ,
- $Q_5 = (2\theta_{B2} - \theta_{A2}, \infty)$ .

Note that  $Q_k$  may be empty for some  $k$ . We use the following result.

**Lemma A6.** Let  $Z = \{0\} \times \mathbb{R}_+$ . Suppose  $u_{it}(x_t) = -(x_t - \theta_{it})^2$  for all  $i \in \{A, B\}$  and  $t$  and  $\theta_{A2} < \theta_{B2}$ . Then  $V_i''(g_1)$  exists and  $V_i''(g_1) \leq 0$  for all  $i \in \{A, B\}$  whenever  $g_1 \in Q_k$  for some  $k = 1, \dots, 5$ .

**Proof.** Consider the second period equilibrium strategies  $\gamma_{A2}^*$  and  $\gamma_{B2}^*$  from Lemma A2. It is easy to see that  $\gamma_{A2}^*(g_1)$  is constant in  $g_1$  on  $Q_1 \cup Q_2 \cup Q_5$ , equal to  $g_1$  on  $Q_3$  and equal to  $2\theta_{B2} - g_1$  on  $Q_4$ . Similarly,  $\gamma_{B2}^*(g_1)$  is constant in  $g_1$  on  $Q_1 \cup Q_4 \cup Q_5$ , equal to  $2\theta_{A2} - g_1$  on  $Q_2$  and equal to  $g_1$  on  $Q_3$ . This implies

$$V_i'(g_1) = \begin{cases} -p_B u'_{i2}(2\theta_{A2} - g_1) & \text{if } g_1 \in Q_2 \\ u'_{i2}(g_1) & \text{if } g_1 \in Q_3 \\ -p_A u'_{i2}(2\theta_{B2} - g_1) & \text{if } g_1 \in Q_4 \\ 0 & \text{if } g_1 \in Q_1 \cup Q_5. \end{cases} \tag{A3}$$

Thus  $V_i''(g_1)$  exists and, by strict concavity of  $u_{i2}$ , we have  $V_i''(g_1) \leq 0$  for all  $i \in \{A, B\}$  and for all  $g_1 \in Q_k$  for  $k \in \{1, \dots, 5\}$ .  $\square$

By Lemma A6,  $V_A$  is continuously differentiable on  $\mathbb{R}_+ \setminus \{2\theta_{A2} - \theta_{B2}, \theta_{A2}, \theta_{B2}, 2\theta_{B2} - \theta_{A2}\}$ . Inspection of (A3) shows that  $V_A(g_1)$  is increasing on  $[0, \theta_{A2}]$ . Since  $f_A(g_1) = u_{A1}(g_1) + \delta V_A(g_1)$  and  $\theta_{A1} > \theta_{A2}$ , we have  $g_A^* > \theta_{A2}$ . Similarly, since  $f_A$  is strictly decreasing on  $Q_5$ , it is not possible to have  $g_A^* \in Q_5$ . From (A3) and  $\theta_{A2} < \theta_{B2}$ , we have

$$\begin{aligned} \lim_{g_1 \rightarrow \theta_{B2}^-} V_A'(g_1) &= u'_{A2}(\theta_{B2}) < 0 < \lim_{g_1 \rightarrow \theta_{B2}^+} V_A'(g_1) = -p_A u'_{A2}(\theta_{B2}), \\ \lim_{g_1 \rightarrow (2\theta_{B2} - \theta_{A2})^-} V_A'(g_1) &= 0 = \lim_{g_1 \rightarrow (2\theta_{B2} - \theta_{A2})^+} V_A'(g_1). \end{aligned} \tag{A4}$$

This implies  $g_A^* \notin \{\theta_{B2}, 2\theta_{B2} - \theta_{A2}\}$ . Thus  $g_A^* \in Q_3 \cup Q_4$  and  $f'_A(g_A^*) = 0$ .

Suppose  $g_{A,k}^*$  satisfies  $f'_A(g_{A,k}^*) = 0$  and  $g_{A,k}^* \in Q_k$  for  $k \in \{3, 4\}$ , then  $g_{A,3}^* = \frac{\theta_{A1} + \delta \theta_{A2}}{1 + \delta}$  and  $g_{A,4}^* = \frac{\theta_{A1} + \delta p_A (2\theta_{B2} - \theta_{A2})}{1 + \delta p_A}$ .

Since  $\theta_{A2} < \theta_{A1}$ , we have  $g_{A,3}^* \in (\theta_{A2}, \theta_{A1})$  and hence  $g_{A,3}^* \in [\theta_{A2}, \theta_{B2}]$ . We need to show that  $g_{A,4}^*$  does not maximize  $f_A$  when  $\theta_{A2} < \theta_{A1} < \theta_{B2}$ . By Lemma A2 part 2 we can evaluate  $f_A$  at  $g_{A,3}^*$  and  $g_{A,4}^*$  and compare these values. We have

$$\begin{aligned} f_A(g_{A,3}^*) &= -\frac{\delta}{1 + \delta} (\theta_{A1} - \theta_{A2})^2, \\ f_A(g_{A,4}^*) &= -\frac{\delta p_A}{1 + \delta p_A} (2\theta_{B2} - \theta_{A1} - \theta_{A2})^2 - \delta(1 - p_A)(\theta_{B2} - \theta_{A2})^2. \end{aligned} \tag{A5}$$

Using (A5),  $f_A(g_{A,4}^*) < f_A(g_{A,3}^*)$  is equivalent to

$$\frac{1}{1 + \delta} (\theta_{A1} - \theta_{A2})^2 < p_A \left( \frac{1}{1 + \delta p_A} \right) (2\theta_{B2} - \theta_{A1} - \theta_{A2})^2 + (1 - p_A)(\theta_{B2} - \theta_{A2})^2. \tag{A6}$$

Note that for  $p_A \in (0, 1)$  and  $\delta \in (0, 1]$  we have  $\frac{1}{1 + \delta} < \frac{1}{1 + \delta p_A} < 1$ . In addition by  $\theta_{A2} < \theta_{A1} < \theta_{B2}$  we have  $(\theta_{A1} - \theta_{A2})^2 < (2\theta_{B2} - \theta_{A1} - \theta_{A2})^2$  and  $(\theta_{A1} - \theta_{A2})^2 < (\theta_{B2} - \theta_{A2})^2$ . Thus the right side of (A6) is a weighted average of two values, each of which is strictly larger than the value on the left side. Hence  $g_{A,3}^*$  is the unique global maximum and is statically Pareto inefficient.

The following lemma completes the proof.

**Lemma A7.** *Let  $Z = \{0\} \times \mathbb{R}_+$ . If  $f_i$  has a unique global maximum at  $g_i^*$  for some  $i \in \{A, B\}$ , then there exists an open interval  $\mathcal{I}$  containing  $g_i^*$  such that if  $g_0 \in \mathcal{I}$ , then  $\gamma_{j1}^*(g_0) \in \mathcal{I}$  for all  $j \in \{A, B\}$ .*

**Proof.** Fix  $i \in \{A, B\}$ . Note that in any equilibrium  $\sigma^*$ , we have  $f_i(\gamma_{j1}^*(g_0)) \geq f_i(g_0)$  for any  $j \in \{A, B\}$  and any initial status quo  $g_0 \in \mathbb{R}_+$  since party  $i$  can always propose  $g_0$  when it is the proposer and can always reject a proposal not equal to  $g_0$  when it is the responder.

Since  $g_i^*$  is the unique global maximum of  $f_i$  and  $f_i$  is continuous, there exists an open interval  $\mathcal{I}$  containing  $g_i^*$  such that if  $g_0 \in \mathcal{I}$  and  $\hat{g}_0 \notin \mathcal{I}$ , then  $f_i(g_0) > f_i(\hat{g}_0)$ . It follows that if  $f_i(\tilde{g}_0) \geq f_i(g_0)$  where  $g_0 \in \mathcal{I}$ , then  $\tilde{g}_0 \in \mathcal{I}$ .

Consider  $g_0 \in \mathcal{I}$ . Suppose party  $i$  is the proposer in period 1. Since  $f_i(\gamma_{i1}^*(g_0)) \geq f_i(g_0)$ , it follows that  $\gamma_{i1}^*(g_0) \in \mathcal{I}$ . Suppose party  $j \neq i$  is the proposer in period 1. Since  $f_i(\gamma_{j1}^*(g_0)) \geq f_i(g_0)$ , it follows that  $\gamma_{j1}^*(g_0) \in \mathcal{I}$ .  $\square$

### Appendix C. Flexible budgetary institutions

The following proposition proves equilibrium existence under any budgetary institution that allows mandatory spending, irrespective of whether discretionary spending is allowed or not.<sup>32</sup>

**Proposition A1.** *Under any budgetary institution that allows mandatory spending, an equilibrium exists.*

**Proof.** We prove equilibrium existence by showing that a solution exists for the proposer’s problem in period 2 given any status quo  $g_1$ , and given this solution, a solution exists for the proposer’s problem in period 1.

Consider the problem of the proposing party  $i \in \{A, B\}$  in the second period under status quo  $g_1 \in \mathbb{R}_+$  and budgetary institution  $Z$  that allows for mandatory spending. The proposing party’s maximization problem is:

$$\begin{aligned} \max_{(k_2, g_2) \in Z} u_{i2}(k_2 + g_2) \\ \text{s.t. } u_{j2}(k_2 + g_2) \geq u_{j2}(g_1). \end{aligned} \tag{P_2}$$

Consider the related problem

$$\max_{x_2 \in A_{j2}(g_1)} u_{i2}(x_2) \tag{P'_2}$$

where  $A_{j2}(g_1) = \{x \in \mathbb{R}_+ | u_{j2}(x) \geq u_{j2}(g_1)\}$  is the responder’s acceptance set under status quo  $g_1$ . If  $\hat{x}_2$  is a solution to  $(P'_2)$ , then any  $(\hat{k}_2, \hat{g}_2) \in Z$  such that  $\hat{k}_2 + \hat{g}_2 = \hat{x}_2$  is a solution to  $(P_2)$ . We use the following properties of  $A_{j2}$ .

**Lemma A8.**  *$A_{j2}(g_1)$  is non-empty, convex and compact for any  $g_1 \in \mathbb{R}_+$  and  $A_{j2}$  is continuous.*

**Proof.** Non-emptiness follows from  $g_1 \in A_{j2}(g_1)$  for all  $g_1 \in \mathbb{R}_+$ . Convexity follows from strict concavity of  $u_{j2}$ . To show compactness, we show that  $A_{j2}(g_1)$  is closed and bounded for all  $g_1 \in \mathbb{R}_+$ . Closedness follows from continuity of  $u_{j2}$ . For boundedness, note that  $u_{j2}$

<sup>32</sup> Equilibrium existence is not immediate because lower hemicontinuity of the second-period acceptance correspondence requires a non-trivial proof.

is differentiable and strictly concave, which implies that  $u_{j2}(x) < u_{j2}(y) + u'_{j2}(y)(x - y)$  for any  $x, y \in \mathbb{R}_+$ . Selecting  $y > \theta_{j2}$  gives  $u'_{j2}(y) < 0$  and taking the limit as  $x \rightarrow \infty$ , we have  $\lim_{x \rightarrow \infty} u_{j2}(x) = -\infty$ . We next establish upper and lower hemicontinuity of  $A_{j2}$  using Lemma A9.

**Lemma A9.** *Let  $X \subseteq \mathbb{R}$  be closed and convex, let  $Y \subseteq \mathbb{R}$  and let  $f : X \rightarrow Y$  be a continuous function. Define  $\varphi : X \rightarrow X$  by*

$$\varphi(x) = \{y \in X \mid f(y) \geq f(x)\}. \tag{A7}$$

1. *If  $\varphi(x)$  is compact  $\forall x \in X$ , then  $\varphi$  is upper hemicontinuous.*
2. *If  $f$  is strictly concave, then  $\varphi$  is lower hemicontinuous.*

**Proof.** To show part 1, since  $\varphi(x)$  is compact for all  $x \in X$ , it suffices to prove that if  $x_n \rightarrow x$  and  $y_n \rightarrow y$  with  $y_n \in \varphi(x_n)$  for all  $n \in \mathbb{N}$ , then  $y \in \varphi(x)$ . Since  $y_n \in \varphi(x_n)$ , we have  $f(y_n) \geq f(x_n)$ . Since  $f$  is continuous,  $x_n \rightarrow x$  and  $y_n \rightarrow y$ , it follows that  $f(y) \geq f(x)$ , hence  $y \in \varphi(x)$ .

To show part 2, fix  $x \in X$ , let  $y \in \varphi(x)$  and consider any  $x_n \rightarrow x$ . We show that there exists a sequence  $y_n \rightarrow y$  and  $n'$  such that  $y_n \in \varphi(x_n)$  for all  $n \geq n'$ . First suppose  $f(y) > f(x)$ . Set  $y_n = y$ . Clearly,  $y_n \rightarrow y$ . By continuity of  $f$ , there exists  $n'$  such that  $f(y_n) \geq f(x_n)$  for all  $n \geq n'$ , that is,  $y_n \in \varphi(x_n)$ . Next suppose  $f(y) = f(x)$ . There are two cases to consider. First, if  $y = x$ , set  $y_n = x_n$ . Clearly  $y_n \rightarrow y$  and  $y_n \in \varphi(x_n)$  for all  $n$ . Second, suppose  $y \neq x$ . By strict concavity of  $f$ , there exist at most one such  $y \in X$ . Set  $y_n = y$  whenever  $f(x_n) \leq f(x)$ . When  $f(x_n) > f(x)$ , by strict concavity of  $f$  and the Intermediate Value Theorem, because  $x_n \rightarrow x$ , there exists  $n'$  such that for all  $n \geq n'$ , there is a unique  $y_n \neq x_n$  such that  $f(y_n) = f(x_n) > f(x)$ . Because  $y_n = y$  whenever  $f(x_n) \leq f(x) = f(y)$  and  $f(y_n) = f(x_n)$  whenever  $f(x_n) > f(x)$ ,  $y_n \in \varphi(x_n)$  for all  $n \geq n'$  and  $y_n \rightarrow y$  follows from continuity of  $f$ .  $\square$

To see that  $A_{j2}$  is upper and lower hemicontinuous, note that it can be written as  $\varphi$  in (A7) with  $X = \mathbb{R}_+$  closed and convex,  $Y = \mathbb{R}$  and  $f = u$  continuous and strictly concave, and we showed before that  $A_{j2}$  is compact-valued.  $\square$

By Lemma A8, for any  $g_1 \in \mathbb{R}_+$ , the acceptance set  $A_{j2}(g_1)$  is non-empty and compact. Applying the Weierstrass's Theorem, a solution exists for  $(P'_2)$ .

We next show that a solution exists to the proposer's problem in period 1. Recall the continuation value  $V_i$  is given by

$$V_i(g_1; \sigma_2^*) = p_A u_{i2}(\kappa_{A2}^*(g_1) + \gamma_{A2}^*(g_1)) + p_B u_{i2}(\kappa_{B2}^*(g_1) + \gamma_{B2}^*(g_1)), \tag{A8}$$

where  $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1)$  is a solution to  $(P'_2)$  for all  $i \in \{A, B\}$ . For any  $g_1 \in \mathbb{R}_+$ , and  $i \in \{A, B\}$ , let  $V_i(g_1) = V_i(g_1; \sigma_2^*)$  and

$$\begin{aligned} F_i(k_1, g_1) &= u_{i1}(k_1 + g_1) + \delta V_i(g_1), \\ f_i(g_1) &= u_{i1}(g_1) + \delta V_i(g_1). \end{aligned} \tag{A9}$$

Lemma A10 establishes some properties of  $V_i$ ,  $F_i$  and  $f_i$ .

**Lemma A10.**  *$V_i$ ,  $F_i$  and  $f_i$  are continuous.  $V_i$  is bounded.*



**Proof.** To show continuity of  $V_i$ , first note that given  $u_{i2}$  is strictly concave, the solution to  $(P'_2)$  is unique for any  $g_1 \in \mathbb{R}_+$ . Since  $A_{j2}$  is non-empty, compact valued and is continuous by Lemma A8, applying the Maximum Theorem we have that the correspondence of maximizers in  $(P'_2)$  is upper hemicontinuous. Since a singleton-valued upper hemicontinuous correspondence is continuous as a function,  $\kappa_{i2}^* + \gamma_{i2}^*$  is continuous. Thus  $V_i(g_1)$  is continuous. Continuity of  $F_i$  and  $f_i$  follow from their definitions and continuity of  $V_i$ .

To show boundedness of  $V_i$ , first note that  $u_{i2}(\kappa_{k2}^*(g_1) + \gamma_{k2}^*(g_1)) \leq u_{i2}(\theta_{i2})$  for all  $k \in \{A, B\}$  and  $g_1 \in \mathbb{R}_+$  because  $\theta_{i2}$  is the unique maximizer of  $u_{i2}$ . Moreover, for  $i \neq j$ , if  $u_{i2}(\kappa_{k2}^*(g_1) + \gamma_{k2}^*(g_1)) < u_{i2}(\theta_{j2})$  for some  $k \in \{A, B\}$  and some  $g_1 \in \mathbb{R}_+$ , then  $k$  could make an alternative proposal that the responder would accept and  $k$  would strictly prefer. It follows that  $u_{i2}(\kappa_{k2}^*(g_1) + \gamma_{k2}^*(g_1)) \geq u_{i2}(\theta_{j2})$ . Thus  $V_i(g_1) \in [u_{i2}(\theta_{j2}), u_{i2}(\theta_{i2})]$  for any  $g_1$ .  $\square$

Fix the initial status quo  $g_0 \in \mathbb{R}_+$ . When only mandatory spending programs are allowed,  $Z = \{0\} \times \mathbb{R}_+$ , and party  $i$ 's equilibrium proposal satisfies  $\kappa_{i1}^*(g_0) = 0$  and

$$\gamma_{i1}^*(g_0) \in \arg \max_{g_1 \in \mathbb{R}_+} f_i(g_1) \text{ s.t. } f_j(g_1) \geq f_j(g_0). \tag{A10}$$

When both types of spending are allowed, we have  $Z = \{(k_t, g_t) \in \mathbb{R} \times \mathbb{R}_+ | k_t + g_t \geq 0\}$ . In equilibrium party  $i$ 's proposal satisfies

$$(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0)) \in \arg \max_{(k_1, g_1) \in Z} F_i(k_1, g_1) \text{ s.t. } F_j(k_1, g_1) \geq F_j(0, g_0). \tag{A11}$$

We show that in each of these problems, the objective function is continuous and the constraint set is compact for any  $g_0 \in \mathbb{R}_+$ . Lemma A10 establishes continuity of  $F_i$  and  $f_i$  and boundedness of  $V_i$ . Compactness follows from an argument analogous to the one made for the second period. Hence for any  $g_0 \in \mathbb{R}_+$ , a solution to each of the optimization problems exists, and therefore an equilibrium exists.  $\square$

### C.1. Proof of Proposition 6

Equilibrium existence follows from Proposition A1. To prove the remainder of the proposition, first, consider the following alternative way of writing the social planner's dynamic problem:

$$\begin{aligned} \max_{(x_1, x_{A2}, x_{B2}) \in \mathbb{R}_+^3} & u_{i1}(x_1) + \delta[p_A u_{i2}(x_{A2}) + p_B u_{i2}(x_{B2})] \\ \text{s.t. } & u_{j1}(x_1) + \delta[p_A u_{j2}(x_{A2}) + p_B u_{j2}(x_{B2})] \geq \bar{U}, \end{aligned} \tag{DSP'}$$

for some  $\bar{U} \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ . The difference between the original social planner's problem (DSP) and the modified social planner's problem (DSP') is that in the modified problem, the social planner is allowed to choose a *distribution* of allocations in period 2. Since the utility functions are concave, it is not optimal for the social planner to randomize and therefore the solution to (DSP) is also the solution to (DSP'). To state this result formally, we denote the solution to (DSP) given  $\bar{U} \in \mathbb{R}$  by  $\mathbf{x}^*(\bar{U}) = (x_1^*(\bar{U}), x_2^*(\bar{U}))$ .

**Lemma A11.** *The solution to the modified social planner's problem (DSP') is  $x_1 = x_1^*(\bar{U})$  and  $x_{A2} = x_{B2} = x_2^*(\bar{U})$ .*

Now fix the initial status quo  $g_0$ . Recall  $f_j(g_0) = u_{j1}(g_0) + \delta V_j(g_0)$  is the responder  $j$ 's status quo payoff. The next result says that the equilibrium mandatory spending in period 1 is the

dynamically Pareto efficient level of spending for period 2 corresponding to  $\bar{U}$ , and the sum of the equilibrium mandatory and discretionary spending is the dynamically Pareto efficient level of spending for period 1 corresponding to  $\bar{U}$ , where  $\bar{U}$  is responder  $j$ 's status quo payoff.

**Lemma A12.** *Let  $Z = \{(k_1, g_1) \in \mathbb{R} \times \mathbb{R}_+ | k_1 + g_1 \geq 0\}$ . For any equilibrium  $\sigma^*$ , given initial status quo  $g_0$ , the equilibrium proposal strategy for party  $i$  in period 1 satisfies  $\gamma_{i1}^*(g_0) = x_2^*(\bar{U})$  and  $\kappa_{i1}^*(g_0) = x_1^*(\bar{U}) - x_2^*(\bar{U})$ , where  $\bar{U} = f_j(g_0)$ .*

**Proof.** If party  $i$  is the proposer in period 1, then party  $i$ 's equilibrium proposal strategy  $(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0))$  is a solution to

$$\begin{aligned} \max_{(k_1, g_1) \in Z} & u_{i1}(k_1 + g_1) + \delta V_i(g_1) \\ \text{s.t. } & u_{j1}(k_1 + g_1) + \delta V_j(g_1) \geq u_{j1}(g_0) + \delta V_j(g_0) \end{aligned} \tag{P1}$$

where  $Z = \{(k_1, g_1) \in \mathbb{R} \times \mathbb{R}_+ | k_1 + g_1 \geq 0\}$ . For notational simplicity we write  $x_1^*$  and  $x_2^*$  instead of  $x_1^*(\bar{U})$  and  $x_2^*(\bar{U})$ . Note that  $(x_1^* - x_2^*, x_2^*)$  is in the feasible set for (P1) since  $x_1^* - x_2^* \in \mathbb{R}$ ,  $x_2^* \in \mathbb{R}_+$  and  $x_1^* - x_2^* + x_2^* \geq 0$ .

We next show that if  $\gamma_{i1}^*(g_0) = x_2^*$  and  $\kappa_{i1}^*(g_0) = x_1^* - x_2^*$ , then the induced equilibrium allocation is  $x_1^*$  in period 1 and  $x_2^*$  in period 2. That  $x_1^{\sigma^*}(g_0) = \gamma_{i1}^*(g_0) + \kappa_{i1}^*(g_0) = x_1^*$  is immediate. That  $x_2^{\sigma^*}(g_0) = x_2^*$  follows since we have  $x_2^* \in [\underline{\theta}_2, \bar{\theta}_2]$ , by Proposition 2 part 1, and  $\kappa_{A2}^*(x_2^*) + \gamma_{A2}^*(x_2^*) = \kappa_{B2}^*(x_2^*) + \gamma_{B2}^*(x_2^*) = x_2^*$  when  $x_2^* \in [\underline{\theta}_2, \bar{\theta}_2]$ , by Lemma A13.

**Lemma A13.** *Let  $Z = \{(k_1, g_1) \in \mathbb{R} \times \mathbb{R}_+ | k_1 + g_1 \geq 0\}$ . Then  $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1) = g_1$  for all  $i \in \{A, B\}$  if  $g_1 \in [\underline{\theta}_2, \bar{\theta}_2]$ .*

**Proof.** Fix the second period proposer  $i$ , responder  $j$  and status quo  $g_1 \in [\underline{\theta}_2, \bar{\theta}_2]$ . In equilibrium we must have  $u_{i2}(\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1)) \geq u_{i2}(g_1)$  and  $u_{j2}(\kappa_{j2}^*(g_1) + \gamma_{j2}^*(g_1)) \geq u_{j2}(g_1)$ . If  $g_1 \in [\underline{\theta}_2, \bar{\theta}_2]$ , then it is statically Pareto efficient and therefore we must have  $u_{i2}(\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1)) = u_{i2}(g_1)$  and  $u_{j2}(\kappa_{j2}^*(g_1) + \gamma_{j2}^*(g_1)) = u_{j2}(g_1)$ , which implies that  $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1) = g_1$ .  $\square$

Finally, we show that  $(x_1^* - x_2^*, x_2^*)$  is the maximizer of (P1). Suppose not. Then proposing  $(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0))$  is better than proposing  $(x_1^* - x_2^*, x_2^*)$ . That is, proposing  $(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0))$  gives proposer  $i$  a higher dynamic payoff while giving the responder  $j$  a dynamic payoff at least as high as  $f_j(g_0)$ . Hence, if  $(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0)) \neq (x_1^* - x_2^*, x_2^*)$ , then the allocation with  $x_1 = \gamma_{i1}^*(g_0) + \kappa_{i1}^*(g_0)$ ,  $x_{A2} = \kappa_{A2}^*(\gamma_{i1}^*(g_0)) + \gamma_{A2}^*(\gamma_{i1}^*(g_0))$ ,  $x_{B2} = \kappa_{B2}^*(\gamma_{i1}^*(g_0)) + \gamma_{B2}^*(\gamma_{i1}^*(g_0))$  does better than  $x_1 = x_1^*$  and  $x_{A2} = x_{B2} = x_2^*$  in (DSP'), which contradicts Lemma A11.  $\square$

By Lemma A12, for status quo  $g_0$  and period-1 proposer  $i \in \{A, B\}$ , the equilibrium allocation is dynamically Pareto efficient since  $x_1^{\sigma^*}(g_0) = \gamma_{i1}^*(g_0) + \kappa_{i1}^*(g_0) = x_1^*(\bar{U})$  and  $x_2^{\sigma^*}(g_0) = \gamma_{i1}^*(g_0) = x_2^*(\bar{U})$  where  $\bar{U} = f_j(g_0)$ . Moreover, given the first-period proposer  $i$ , the period-2 equilibrium level of spending equals  $\gamma_{i1}^*(g_0)$  and hence is independent of the period-2 proposer. Hence  $\sigma^*$  is a dynamically Pareto efficient equilibrium given  $g_0$ .  $\square$

### C.2. Proof of Proposition 7

We prove part 1 by contradiction. Suppose  $x_t^* \neq x_{t'}^*$  for some  $t \neq t'$ . Then there exists  $s \in S$  such that  $x_t^*(s) \neq x_{t'}^*(s)$ . Without loss of generality, assume  $x_t^*(s) < x_{t'}^*(s)$ .

From strict concavity of  $u_i$  for all  $i \in \{A, B\}$ , we have  $\alpha u_i(x_t^*(s), s) + (1 - \alpha)u_i(x_t'^*(s), s) < u_i(\alpha x_t^*(s) + (1 - \alpha)x_t'^*(s), s)$  for any  $\alpha \in (0, 1)$ . Let  $\alpha = \frac{\delta^{t-1}}{\delta^{t-1} + \delta^{t'-1}} \in (0, 1)$  and  $x' = \alpha x_t^*(s) + (1 - \alpha)x_t'^*(s)$ , we have

$$\delta^{t-1}u_i(x_t^*(s), s) + \delta^{t'-1}u_i(x_t'^*(s), s) < (\delta^{t-1} + \delta^{t'-1})u_i(x', s) \tag{A12}$$

for all  $i \in \{A, B\}$ , which contradicts that  $\mathbf{x}^*$  is a solution to (DSP-S).

Next we prove part 2. Fix  $i, j \in \{A, B\}$  with  $i \neq j$ . For any  $\bar{U} > \sum_{t=1}^T \delta^{t-1} \mathbb{E}_s[u_j(\theta_{js}, s)]$ , (DSP-S) has no solution, so assume  $\bar{U} \leq \sum_{t=1}^T \delta^{t-1} \mathbb{E}_s[u_j(\theta_{js}, s)]$ .

For  $\bar{U} = \sum_{t=1}^T \delta^{t-1} \mathbb{E}_s[u_j(\theta_{js}, s)]$ , the solution to (DSP-S) is  $x_t^*(s) = \theta_{js}$  for all  $t$  and  $s \in S$  and for any  $\bar{U} \leq \sum_{t=1}^T \delta^{t-1} \mathbb{E}_s[u_j(\theta_{is}, s)]$ , the solution to (DSP-S) is  $x_t^*(s) = \theta_{is}$  for all  $t$  and  $s \in S$ . What remains is the case when  $\bar{U} \in (\sum_{t=1}^T \delta^{t-1} \mathbb{E}_s[u_j(\theta_{is}, s)], \sum_{t=1}^T \delta^{t-1} \mathbb{E}_s[u_j(\theta_{js}, s)])$ . From the Lagrangian for (DSP-S), the first order necessary condition with respect to  $x_t(s)$  for any  $t$  and  $s \in S$  is  $\delta^{t-1}u'_i(x_t^*(s), s) + \lambda^* \delta^{t-1}u'_j(x_t^*(s), s) = 0$  for some  $\lambda^* > 0$ . If  $\theta_{As} \neq \theta_{Bs}$ , then this condition simplifies to  $-\frac{u'_i(x_t^*(s), s)}{u'_j(x_t^*(s), s)} = \lambda^*$ . If  $\theta_{As} = \theta_{Bs}$ , then  $x_t^*(s) = \theta_{As} = \theta_{Bs}$ .  $\square$

### C.3. Proof of Proposition 8

Suppose the state in period 1 is  $s_1$ . Consider the following problem:

$$\begin{aligned} \max_{\{x_t: S \rightarrow \mathbb{R}_+\}_{t=1}^T} & u_i(x_1(s_1), s_1) + \sum_{t=2}^T \delta^{t-1} \mathbb{E}_s[u_i(x_t(s), s)] \\ \text{s.t.} & u_j(x_1(s_1), s_1) + \sum_{t=2}^T \delta^{t-1} \mathbb{E}_s[u_j(x_t(s), s)] \geq \bar{U}' \end{aligned} \tag{DSP-S'}$$

for some  $\bar{U}' \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ .

The difference between (DSP-S') and (DSP-S) is that  $x_1(s)$  for  $s \in S \setminus \{s_1\}$  does not enter (DSP-S'), so the solution to (DSP-S') does not pin down  $x_1(s)$  for  $s \in S \setminus \{s_1\}$ .

Analogous to the proof of Proposition 6, consider the following alternative way of writing the social planner's problem:

$$\begin{aligned} \max_{\{x_t^A, x_t^B: S \rightarrow \mathbb{R}_+\}_{t=1}^T} & u_i(x_1^i(s_1), s_1) + \sum_{t=2}^T \delta^{t-1} \mathbb{E}_s[p_A u_i(x_t^A(s), s) + p_B u_i(x_t^B(s), s)] \\ \text{s.t.} & u_j(x_1^i(s_1), s_1) + \sum_{t=2}^T \delta^{t-1} \mathbb{E}_s[p_A u_j(x_t^A(s), s) + p_B u_j(x_t^B(s), s)] \geq \bar{U}' \end{aligned} \tag{DSP-S''}$$

for some  $\bar{U}' \in \mathbb{R}$ ,  $i, j \in \{A, B\}$  and  $i \neq j$ .

Since  $u_A$  and  $u_B$  are strictly concave in  $x$  for all  $s$ , clearly any solution to (DSP-S'') satisfies  $x_t^A(s) = x_t^B(s)$  for all  $t$  and  $s$ . So we can just consider (DSP-S').

**Lemma A14.** *If  $\mathbf{x}$  is a solution to (DSP-S'), then for any  $t, t' \geq 2$ ,  $x_t = x_{t'}$ . Moreover,  $x_1(s_1) = x_t(s_1)$  for  $t \geq 2$ .*

The proof of Lemma A14 is immediate from the proof of Proposition 7. We then have the following result.

**Lemma A15.** *If  $\mathbf{x}$  is a solution to (DSP-S) for some  $\bar{U}$ , then it is a solution to (DSP-S') for some  $\bar{U}'$ . If  $\mathbf{x}$  is a solution to (DSP-S') for some  $\bar{U}'$  and it satisfies that  $x_1(s) = x_t(s)$  for  $t \geq 2$  and for all  $s$ , then  $\mathbf{x}$  is a solution to (DSP-S) for some  $\bar{U}$ .*

**Proof.** Fix  $i, j \in \{A, B\}$  with  $i \neq j$  and  $s_1 \in S$  and let  $p_1$  be the probability distribution of  $s$  in period 1. First, note that  $x_1(s)$  for any  $s \in S \setminus \{s_1\}$  does not enter either the objective function or the constraint in (DSP-S'). Hence if  $\mathbf{x}$  is a solution to (DSP-S) with  $\bar{U}$ , then  $\mathbf{x}$  is a solution to (DSP-S') with  $\bar{U}' = \bar{U} + (1 - p_1(s_1))u_j(x_1(s_1), s_1) - \sum_{s \in S \setminus \{s_1\}} p_1(s)u_j(x_1(s), s)$ . Second, note that by Proposition 7, if  $\mathbf{x} = \{x_t\}_{t=1}^T$  is a solution to (DSP-S), then  $x_t = x_{t'}$  for any  $t$  and  $t'$ . Hence, if  $\mathbf{x}$  with  $x_1(s) = x_t(s)$  for  $t \geq 2$  and for all  $s \in S$  solves (DSP-S') with  $\bar{U}'$ , then  $\mathbf{x}$  is a solution to (DSP-S) with  $\bar{U} = \bar{U}' - (1 - p_1(s_1))u_j(x_1(s_1), s_1) + \sum_{s \in S \setminus \{s_1\}} p_1(s)u_j(x_1(s), s)$ .  $\square$

We prove Proposition 8 by establishing Lemmas A16 and A17 below. With slight abuse of terminology, we call a spending rule  $g \in \mathcal{M}$  dynamically Pareto efficient if  $\{g_t\}_{t=1}^T$  with  $g_t = g$  for all  $t$  is a dynamically Pareto efficient allocation rule.

**Lemma A16.** For any  $t$ , if the status quo  $g_{t-1}$  is dynamically Pareto efficient, then  $\gamma_{it}^*(g_{t-1}, s_t) = g_{t-1}$  for all  $s_t \in S$  and all  $i \in \{A, B\}$ .

**Proof.** Suppose the state in period  $t$  is  $s_t$ . For any status quo  $g_{t-1}$  in period  $t$ , the proposer  $i$ 's equilibrium continuation payoff is weakly higher than  $u_i(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \times \mathbb{E}_s[u_i(g_{t-1}(s), s)]$  and the responder  $j$ 's equilibrium continuation payoff is weakly higher than  $u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s)]$ . To see this, note that for any status quo in any period, a responder accepts a proposal if it is the same as the status quo, implying that a proposer can maintain the status quo by proposing it. Hence, proposer  $i$  can achieve the payoff above by proposing to maintain the status quo in period  $t$  and in future periods continue to propose to maintain the status quo if it is the proposer and rejects any proposal other than the status quo if it is the responder. Similarly, responder  $j$  can achieve the payoff above by rejecting any proposal other than the status quo in period  $t$  and in future periods continue to reject any proposal other than the status quo if it is the responder and propose to maintain the status quo if it is the proposer.

Consider proposer  $i$ 's problem in period  $t$

$$\begin{aligned} & \max_{g_t \in \mathcal{M}} u_i(g_t(s_t), s_t) + \delta V_{it}(g_t; \sigma^*) \\ & \text{s.t. } u_j(g_t(s_t), s_t) + \delta V_{jt}(g_t; \sigma^*) \geq u_j(g_{t-1}(s_t), s_t) + \delta V_{jt}(g_{t-1}; \sigma^*), \end{aligned}$$

where  $V_{it}(g; \sigma^*)$  is the expected discounted utility of party  $i \in \{A, B\}$  in period  $t$  generated by strategies  $\sigma^*$  when the status quo is  $g$ . As shown in the previous paragraph,  $u_j(g_{t-1}(s_t), s_t) + \delta V_{jt}(g_{t-1}, \sigma^*) \geq u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s)]$ .

Suppose the solution to the proposer's problem in period  $t$  is  $g_t^* \neq g_{t-1}$ . Then there exists an allocation rule with  $x_t = g_t^*$  and future spending rules induced by status quo  $g_t^*$  and equilibrium  $\sigma^*$  such that party  $i$ 's dynamic payoff is higher than  $u_i(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_i(g_{t-1}(s), s)]$  and party  $j$ 's dynamic payoff is higher than  $u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s)]$ . But if  $g_{t-1}$  is dynamically Pareto efficient, then having the spending rule in all periods  $t' \geq t$  equal to  $g_{t-1}$  is a solution to (DSP-S') with  $\bar{U}' = u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s)]$ , a contradiction.  $\square$

**Lemma A17.** For any initial status quo  $g_0$  and any  $s_1 \in S$ , the proposer makes a proposal in period 1 that is dynamically Pareto efficient, that is,  $\gamma_{i1}^*(g_0, s_1)$  is dynamically Pareto efficient for all  $i \in \{A, B\}$ .

**Proof.** Fix  $g_0$  and  $s_1$ . Let  $f_j(g_0, s_1)$  be the responder  $j$ 's status quo payoff. That is,

$$f_j(g_0, s_1) = u_j(g_0(s_1), s_1) + \delta V_{j1}(g_0; \sigma^*).$$

Let  $\bar{U}' = f_j(g_0, s_1)$  and denote the solution to (DSP-S') by  $\mathbf{x}(\bar{U}') = (x_1(\bar{U}'), \dots, x_T(\bar{U}'))$ . By Lemma A14,  $x_t(\bar{U}') = x_{t'}(\bar{U}')$  for any  $t, t' \geq 2$  and  $x_1(\bar{U}')(s_1) = x_t(\bar{U}')(s_1)$  for  $t \geq 2$ . Without loss of generality, suppose  $\mathbf{x}(\bar{U}')$  satisfies  $x_1(\bar{U}') = x_t(\bar{U}')$  for  $t \geq 2$ . Note that  $x_1(\bar{U}')$  is a dynamically Pareto efficient spending rule by Lemma A15.

We next show that  $\gamma_{i1}^*(g_0, s_1) = x_1(\bar{U}')$ . First note that if  $\gamma_{i1}^*(g_0, s_1) = x_1(\bar{U}')$ , then, since  $x_1(\bar{U}')$  is dynamically Pareto efficient, the induced equilibrium allocation is  $\mathbf{x}(\bar{U}')$  by Lemma A16. We show by contradiction that  $\gamma_{i1}^*(g_0, s_1) = x_1(\bar{U}')$  is the solution to the proposer's problem. Suppose not. Then proposing  $\gamma_{i1}^*(g_0, s_1)$  is strictly better than proposing  $x_1(\bar{U}')$ , that is, proposing  $\gamma_{i1}^*(g_0, s_1)$  gives  $i$  a strictly higher dynamic payoff while giving  $j$  a dynamic payoff at least as high as  $f_j(g_0, s_1)$ . But since  $\mathbf{x}(\bar{U}')$  is a solution to (DSP-S') and hence a solution to (DSP-S''), this is a contradiction.  $\square$

## References

- Acemoglu, D., Golosov, M., Tsyvinski, A., 2008. Political economy of mechanisms. *Econometrica* 76 (3), 619–641.
- Acemoglu, D., Golosov, M., Tsyvinski, A., 2011. Power fluctuations and political economy. *J. Econ. Theory* 146 (3), 1009–1041.
- Aguiar, M., Amador, M., 2011. Growth in the shadow of expropriation. *Q. J. Econ.* 126 (2), 651–697.
- Alesina, A., Perotti, R., 1995. The political economy of budget deficits. *IMF Staff Pap.* 42 (1), 1–31.
- Alesina, A., Tabellini, G., 1990. A positive theory of fiscal deficits and government debt. *Rev. Econ. Stud.* 57 (3), 403–414.
- Anesi, V., Seidmann, D.J., 2015. Bargaining in standing committees with an endogenous default. *Rev. Econ. Stud.* 82 (3), 825–867.
- Azzimonti, M., 2011. Barriers to investment in polarized societies. *Am. Econ. Rev.* 101 (5), 2182–2204.
- Azzimonti, M., Battaglini, M., Coate, S., 2016. The costs and benefits of balanced budget rules: lessons from a political economy model of fiscal policy. *J. Public Econ.* 136, 45–61.
- Bai, J., Lagunoff, R., 2011. On the 'Faustian' dynamics of policy and political power. *Rev. Econ. Stud.* 78 (1), 17–48.
- Baron, D.P., 1996. A dynamic theory of collective goods programs. *Am. Polit. Sci. Rev.* 90 (2), 316–330.
- Baron, D.P., Bowen, T.R., 2014. Dynamic Coalitions. Working Paper.
- Battaglini, M., Coate, S., 2007. Inefficiency in legislative policy-making: a dynamic analysis. *Am. Econ. Rev.* 97 (1), 118–149.
- Battaglini, M., Palfrey, T., 2012. The dynamics of distributive politics. *Econ. Theory* 49 (3), 739–777.
- Besley, T., Coate, S., 1998. Sources of inefficiency in a representative democracy: a dynamic analysis. *Am. Econ. Rev.* 88 (1), 139–156.
- Besley, T., Coate, S., 2003. Centralized versus decentralized provision of local public goods: a political economy approach. *J. Public Econ.* 87 (12), 2611–2637.
- Besley, T., Smart, M., 2007. Fiscal restraints and voter welfare. *J. Public Econ.* 91 (3–4), 755–773.
- Bierbrauer, F.J., Boyer, P.C., 2016. Efficiency, welfare, and political competition. *Q. J. Econ.* 131 (1), 461–518.
- Bowen, T.R., Chen, Y., Eraslan, H., 2014. Mandatory versus discretionary spending: the status quo effect. *Am. Econ. Rev.* 104 (10), 2941–2974.
- Bowen, T.R., Chen, Y., Eraslan, H., Zápal, J., 2016. Efficiency of flexible budgetary institutions. NBER Work. Pap. Ser. 22457.
- Breasted, J.H., 1945. *A History of Egypt*. Charles Scribner's Sons, New York, NY.
- Budget of the United States Government, 2015. Budget of the United States Government, Fiscal year 2015, Historical Tables and Public Budget Database. <http://www.gpo.gov/fdsys/browse/collectionGPO.action?collectionCode=BUDGET>.
- Caballero, R.J., Yared, P., 2010. Future rent-seeking and current public savings. *J. Int. Econ.* 82 (2), 124–136.

- Callander, S., Krehbiel, K., 2014. Gridlock and delegation in a changing world. *Am. J. Polit. Sci.* 58 (4), 819–834.
- Canadian Minister of Justice, 2014. Employment Insurance Act. <http://laws-lois.justice.gc.ca/pdf/e-5.6.pdf>.
- Chen, Y., Eraslan, H., 2016. Dynamic agenda setting. *Am. Econ. J. Microecon.*, forthcoming. <https://www.aeaweb.org/articles?id=10.1257/mic.20150298>.
- Diermeier, D., Egorov, G., Sonin, K., 2013. Endogenous property rights. NBER Work. Pap. Ser. 19734.
- Diermeier, D., Fong, P., 2011. Legislative bargaining with reconsideration. *Q. J. Econ.* 126 (2), 947–985.
- Dixit, A., Grossman, G.M., Gul, F., 2000. The dynamics of political compromise. *J. Polit. Econ.* 108 (3), 531–568.
- Duggan, J., Kalandrakis, A., 2012. Dynamic legislative policy making. *J. Econ. Theory* 147 (5), 1653–1688.
- Dziuda, W., Loeper, A., 2016. Dynamic collective choice with endogenous status quo. *J. Polit. Econ.* 124 (4), 1148–1186.
- Forand, J.G., 2014. Two-party competition with persistent policies. *J. Econ. Theory* 152, 64–91.
- Halac, M., Yared, P., 2014. Fiscal rules and discretion under persistent shocks. *Econometrica* 82 (5), 1557–1614.
- Hallerberg, M., Strauch, R.R., von Hagen, J., 2009. *Fiscal Governance in Europe*. Cambridge University Press, Cambridge.
- House Budget Committee, 2014. The path to prosperity: fiscal year 2015 budget resolution. <http://budget.house.gov/fy2015/>.
- Kalandrakis, A., 2004. A three-player dynamic majoritarian bargaining game. *J. Econ. Theory* 116 (2), 294–322.
- Kalandrakis, A., 2010. Minimum winning coalitions and endogenous status quo. *Int. J. Game Theory* 39 (4), 617–643.
- Kalandrakis, A., 2016. Pareto efficiency in the dynamic one-dimensional bargaining model. *J. Theor. Polit.* 28 (4), 525–536.
- Krusell, P., Ríos-Rull, J.-V., 1996. Vested interests in a positive theory of stagnation and growth. *Rev. Econ. Stud.* 63 (2), 301–329.
- Levy, G., Razin, R., 2013. Dynamic legislative decision making when interest groups control the agenda. *J. Econ. Theory* 148 (5), 1862–1890.
- Lizzeri, A., Persico, N., 2001. The provision of public goods under alternative electoral incentives. *Am. Econ. Rev.* 91 (1), 225–239.
- Ma, Y., 2014. Curse or Blessing? On the Welfare Consequences of Divided Government and Policy Gridlock. Working Paper.
- Mas-Colell, A., Whinston, M.D., Green, J.R., 1995. *Microeconomic Theory*. Oxford University Press, Oxford.
- Nicholson, W., Needels, K., 2006. Unemployment insurance: strengthening the relationship between theory and policy. *J. Econ. Perspect.* 20 (3), 47–70.
- Nunnari, S., Zápál, J., 2015. Dynamic Elections and Ideological Polarization. Working Paper.
- OECD, 2012. International budget practices and procedures database. <http://www.oecd.org/gov/budgeting/>.
- Persson, T., Svensson, L.E.O., 1984. Time-consistent fiscal policy and government cash-flow. *J. Monet. Econ.* 14 (3), 365–374.
- Persson, T., Tabellini, G., 1996a. Federal fiscal constitutions: risk sharing and moral hazard. *Econometrica* 64 (3), 623–646.
- Persson, T., Tabellini, G., 1996b. Federal fiscal constitutions: risk sharing and redistribution. *J. Polit. Econ.* 104 (5), 979–1009.
- Piguillem, F., Riboni, A., 2012. Dynamic Bargaining over Redistribution in Legislatures. Working Paper.
- Piguillem, F., Riboni, A., 2015. Spending-biased legislators: discipline through disagreement. *Q. J. Econ.* 130 (2), 901–949.
- Riboni, A., Ruge-Murcia, F.J., 2008. The dynamic (in)efficiency of monetary policy by committee. *J. Money Credit Bank.* 40 (5), 1001–1032.
- Romer, T., Rosenthal, H., 1978. Political resource allocation, controlled agendas, and the status quo. *Public Choice* 33 (48), 27–43.
- Romer, T., Rosenthal, H., 1979. Bureaucrats versus voters: on the political economy of resource allocation by direct democracy. *Q. J. Econ.* 93 (4), 563–587.
- Santiso, C., 2006. Budget Institutions and Fiscal Responsibility: Parliaments and the Political Economy of the Budget Process. World Bank Institute Working Paper Series 35863.
- Shah, A., 2007. *Budgeting and Budgetary Institutions*. The World Bank, Washington, DC.
- Stockman, D.R., 2001. Balanced-budget rules: welfare loss and optimal policies. *Rev. Econ. Dyn.* 4 (2), 438–459.
- United States Department of Labor, 2015. Fiscal year 2015 Department of Labor budget in brief. <http://www.dol.gov/dol/budget/2015/pdf/fy2015bib.pdf>.
- U.S. House of Representatives, Office of the Legislative Counsel, 2013. Federal-state extended unemployment compensation act of 1970. <http://legcounsel.house.gov/Comps/fseuca70.pdf>.

Van Weelden, R., 2013. Candidates, credibility, and re-election incentives. *Rev. Econ. Stud.* 80 (4), 1622–1651.

Yared, P., 2010. Politicians, taxes and debt. *Rev. Econ. Stud.* 77 (2), 806–840.

Zápal, J., 2011. Explicit and Implicit Status-Quo Determination in Dynamic Bargaining: Theory and Application to FOMC Directive. Working Paper.